The Beetle in the Box: Exploring IF-Dialogues

Shahid Rahman * Cédric Dégremont †

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> Suppose everyone had a box with something in it: call it "beetle." No one can look into anyone else's box, and everyone says he knows what a beetle is only by looking at his *beetle*. ... The thing in the box has no place in the language-game at all ...
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> Ludwig Wittgenstein, *Philosophical Investigations* I, sec. 293.[1]

> It is especially important to realize that Wittgenstein does not mean by the last quoted sentence that the private object (private experience) does not in fact play any role in the language. On the contrary, Wittgenstein is saying that it is irrelevant only on the mistaken assumption that words like "beetle" could operate as mere designations independent of any language-game. ... Now, Wittgenstein might very well have believed that language is in fact social in nature. What Wittgenstein argues is nevertheless that language is an essentially public enterprise in the sense of being publicly accessible.
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> Jaakko Hintikka, *On Wittgenstein*, pp. 42-43.

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*UFR de Philosophie, Université Lille 3: shahid.rahman@univ-lille3.fr

†UFR de Philosophie, Université Lille 3: cedric.degremont@etu.univ-lille3.fr

[1] Cheryl Lobb de Rahman suggested that this passage of the *Philosophical Investigations* evokes the well-known children English rhyme *Alexander Beetle* of the late twenties by Alan Alexander Milne (1882-1956) which might have inspired Wittgenstein's example:

I had a little beetle
So that beetle was his name
I called him Alexander
And he answered just the same
And I put him in a matchbox
...
But it is difficult to catch
An excited sort of beetle
You've mistaken for a match
...

1 Motivation and Aims

The fact that game-theoretical semantics (GTS) and dialogic are sisters has been widely acknowledged. The differences between the original approaches have been discussed too: while GTS relates to the study of *truth in a model*, dialogic has explored the possibilities of a certain type of proof-theoretical approach to *validity*. Despite close relations between the two approaches, no thorough analysis of their interaction has yet been undertaken before.

In a recent paper by Shahid Rahman and Tero Tulenheimo ([Rahman & Tulenheimo 2005]) this task has been carried out systematically. More precisely, Rahman and Tulenheimo worked out for classical propositional logic and classical first-order logic an exact connection between 'intuitionistic dialogues with hypotheses' and semantical games. Basically, it is shown there that the existence of a winning strategy for one of the players (called *Proponent*) in a dialogue $\mathcal{D}(A; H_1, \ldots, H_n)$ corresponding to a sentence A with a finite number of hypotheses H_i expressing those versions of the third excluded which are relevant to that sentence, gives rise to a family of *Eloise*'s winning strategies in semantical games $G(A, M)$, one strategy for each model M; and, conversely, it is shown how to construct a winning strategy for *Proponent* in the dialogue $\mathcal{D}(A; H_1, \ldots, H_n)$ out of *Eloise*'s winning strategies in games $G(A, M)$. The proofs are constructive in the sense that it is explicitly shown how strategy for one type of game is built using a strategy for the other type of game. One corollary of the paper is that each disjunct of the hypotheses of the dialogue at stake yields the dialogical version of truth in a model.

The aim of this paper is to furnish the first exploratory steps towards IF-Logic with GTS semantics. One of the purposes of such a study is to extend the results of the paper mentioned above in order to build tableaux proofs for some fragments of IF. However in these first steps we will center our developments in the dialogical version of truth in a model rather than in the notion of validity. We will present here two dialogical reconstructions of IF, one much more in the spirit of the GTS version of IF and the second which stems mostly of an idea of the second author of the paper, is in a much more work in progress phase. We call, the latter reconstruction *Poker IF-Dialogues*, which seems to be related to information delay rather than to imperfect transmission of information.

Despite the differences between these two different dialogical approaches to IF-Logic the first author claims that both might be thought as implementing Hintikka's challenging interpretation of Wittgenstein's private-language-argument quoted above. More precisely the claim is that IF-Logic and in our case IF-Dialogues display the process involved in those language-games where private information does play a role. Indeed, in our dialogues we use *private moves*, by means of which information is encoded in a *box*, and *public moves* including those that open the box. Unfortunately we cannot work out this philosophical claim here and we will leave it for a forthcoming paper, but we could not resist the temptation to at least mention the central idea behind our approaches.

2 Introduction: FOL and Dialogues

Let us introduce the languages we will be working with. We will consider the languages of Propositional logic, First-order logic and IF First-order logic.

We consider Propositional logic (**PL**) the syntax of which is defined with help of a countable set **prop** of propositional letters $p, q, \ldots$, with the usual connectives, the corresponding brackets and with the standard definition of well-formed formula.

The first-order syntax is defined on a given finite set τ of symbols consisting of a set *Const* of constants $c_0, c_1, \ldots$, a set *Relations* of relation symbols $\mathrm{R}_0, \mathrm{R}_1, \ldots$ of various arities, and a set $Var = \{x_0, x_1, \ldots\}$ of individual variables. Let us refer to constants and variables as *terms* $t_0, t_1, \ldots$ with $Terms = Var \cup Const$ As usual, we will use the existential quantifier ($\exists x$) and the universal quantifier ($\forall x$). These will yield the known notion of well-formed formula of **FOL**

We also assume the standard semantics of FOL

2.1 Dialogical First-Order Logic

2.1.1 Formal dialogues

Let us reconstruct in dialogical terms the notion of validity in First-order logic (the present formulation stems from [Rahman & Tulenheimo 2005], for a somewhat different account, see [Rahman & Keiff 2005].) We first define a language $\mathcal{L}[\tau]$; this language will basically be obtained from First-order logic (of vocabulary τ) by adding certain metalogical symbols. For the sake of fuller exposition we consider First-order logic with implication, or **FO**$[\rightarrow, \tau]$.

We introduce special *force symbols* ? and !. An *expression* of $\mathcal{L}[\tau]$ is either a formula of **FO**$[\rightarrow, \tau]$, or one of the following strings:

$$L, R, \vee, \forall x_i/c_j \text{ or } \exists x_i,$$

where x_i is any variable and c_j any constant. We refer to the latter type of expressions as *attack markers*. In addition to expressions and force symbols, for $\mathcal{L}[\tau]$ we have available *labels* **O** and **P**, standing for the players (*Proponent*, *Opponent*) of dialogues. We will refer to **P** as 'she' and to **O** as 'he'. Every expression e of $\mathcal{L}[\tau]$ can be augmented with labels **P** or **O** on the one hand, and by force symbols ? and ! on the other, so as to yield the strings

P-!-e, **O**-!-e, **P**-?-e and **O**-?-e.

These strings are said to be *(dialogically) signed expressions*. Their role is to signify that in the course of a dialogue, the move corresponding to the expression e is to be made by **P** or **O**, respectively, and that the move is made as a defense (!) or an attack (?). We will use X and Y as variables for **P** and **O**, always assuming $X \neq Y$.

2.1.2 Particle rules

An *argumentation form* or *particle rule* is an abstract description of the way a formula, according to its outmost form, can be criticized, and how to answer the critique. It is abstract in the sense that this description can be carried out without reference to a specified context. In dialogical logic, these rules are said to state the *local semantics*, for they show how the game runs locally: what is at stake is only the critique and the answer corresponding to a given logical constant, rather than the whole context where the logical constant is embedded.[2] The particle rules fix the dialogical semantics of the logical constants of $\mathcal{L}[\tau]$ in the following way:

[2] There can be no particle rule corresponding to atomic formulas. But it is possible to add a set of *Opponent*'s initial concessions to the particle rules. This is done in 'material dialogues' explained in section 2.2.

	$\wedge$	$\vee$	$\rightarrow$
Assertion	X-!-$A \wedge B$	X-!-$A \vee B$	X-!-$A \rightarrow B$
attack	Y-?-L or Y-?-R	Y-?-$\vee$	Y-!-A
defense	X-!-A *resp.* X-!-B	X-!-A or X-!-B	X-!-B

	$\forall$	$\exists$	$\neg$
Assertion	X-!-$\forall xA$	X-!-$\exists xA$	X-!-$\neg A$
attack	Y-?-$\forall x/c$ for any c available to Y	Y-?-$\exists x$	Y-!-A
defense	X-!-$A[x/c]$ for any c chosen earlier by Y	X-!-$A[x/c]$ for any c available to X	–

In the diagram, $A[x/c]$ stands for the result of substituting the constant c for every occurrence of the variable x in the formula A.

A more thorough way to stress the sense in which the particle rules determine local semantics is to see these rules as defining *state* of a (structurally not yet determined) game.

Definition 2.1 (State of a dialogue) *Let $A \in \mathbf{FO}[\rightarrow, \tau]$, and let a countable set $\{c_0, c_1, \ldots\}$ of individual constants be fixed. A* state of the dialogue $\mathcal{D}(A)$ *corresponding to the formula A is a quintuple $\langle B, X, f, e, \sigma\rangle$ such that:*

- *B is a (proper or improper) subformula of A.*
- *X-f-e is a dialogically signed expression: $X \in \{\mathbf{O}, \mathbf{P}\}$, $f \in \{?, !\}$, and $e \in \mathcal{L}[\tau]$.*
- *$\sigma : Free[B] \longrightarrow \{c_0, c_1, \ldots\}$ is a function mapping the free variables of B to individual constants.*

The component e is either a formula of $\mathbf{FO}[\tau]$, or an attack marker. We stipulate that in the former case, always $e = B$.

Given a force f, let us write f' for the opposite force, i.e. let $f' \in \{?, !\} \setminus \{f\}$. Each state $\langle B, X, f, e, \sigma\rangle$ has an associated *role assignment*, indicating which player occupies the role of *Challenger* (?) and which the role of *Defender* (!). The role assignment is a function $\rho : \{\mathbf{P}, \mathbf{O}\} \longrightarrow \{?, !\}$ such that $\rho(X) = f$ and $\rho(Y) = f'$.

State $\langle B_2, X_2, f_2, e_2, \sigma_2\rangle$ is *reachable* from state $\langle B_1, X_1, f_1, e_1, \sigma_1\rangle$, if it is a result of X_1 making a move in accordance with the appropriate particle rule in the role f_1. If the role is that of *Challenger* ($f_1 = ?$), the player states an attack, whereas if the role is that of *Defender* ($f_1 = !$), the player poses a defense.

Let us take a closer look at the transitions from one state to another. Particle rules determine which state S_2 of a dialogue is reachable from a given other state S_1. Notice that the player who defends need not be the same at both states. In order for state S_2 to be reachable from state $S_1 = \langle B, X, f, e, \sigma\rangle$, it must satisfy the following.

- *Particle rule for negation:* If $B = e$, $f = !$ and B is of the form $\neg C$, then

  $$S_2 = \langle C, Y, !, C, \sigma\rangle.$$

  So if $\mathbf{P}$ is *Defender* of $\neg C$ at S_1, then $\mathbf{O}$ is *Defender* of C at S_2, and $\mathbf{P}$ will challenge (counterattack) C; and dually, if $\mathbf{P}$ is *Challenger* of $\neg C$ at S_1.

Notice that here state S_2 involves the claim that C can be defended; however, this claim has been asserted in the course of an attack, and the whole move from S_1 to S_2 counts as an attack on the initial negated formula, i.e. an attack on C. Actually this follows from the fact that at S_2, the roles of the players are inverted as compared with S_1. Counterattack may yield from S_2 a further state, $S_3 = \langle C, X, ?, *, \sigma\rangle$, where C is the formula considered, and the attack pertains to the relevant logical constant of C, for which $*$ is a suitable attack marker.

- *Particle rule for conjunction:* If $B = e$, $f = !$ and B is of the form $C \wedge D$, then

$$S_2 = \langle C, X, !, C, \sigma\rangle \quad \text{or} \quad S_2 = \langle D, X, !, D, \sigma\rangle,$$

  according to the choice of *Challenger* between the attacks ?-L and ?-R. (Here *Challenger* is Y: Y's role is ? here.)
- *Particle rule for disjunction:* If $B = e$, $f = !$ and B is of the form $C \vee D$, then

$$S_2 = \langle C, X, !, C, \sigma\rangle \quad \text{or} \quad S_2 = \langle D, X, !, D, \sigma\rangle,$$

  according to the choice of *Defender*, reacting to the attack ?-$\vee$ of *Challenger*. (Here *Defender* is X: X's role is ! here.)
- *Particle rule for implication:* If $B = e$, $f = !$ and B is of the form $C \rightarrow D$, then

$$S_2 = \langle C, Y, !, C, \sigma\rangle$$

  and, further, state

$$S_3 = \langle D, X, !, D, \sigma\rangle$$

  is reachable from S_2. So if **P** is *Defender* of $C \rightarrow D$ at S_1, and hence **O** is *Defender* of C at S_2, it is **P** who will be *Defender* of D at S_3.

To attack an implication amounts to being prepared to defend its antecedent, and so it should be noticed that the defense of C at state S_2 counts as an attack. If **P** is *Defender* of $C \rightarrow D$ at S_1, at state S_3 reachable from S_2, either **P** may defend D, or else **P** may counterattack C, thus yielding a further state, $S_4 = \langle C, X, ?, *, \sigma\rangle$, where C is the formula considered, and the relevant logical particle of C is attacked, $*$ being an appropriate attack marker.

- *Particle rule for universal quantifier:* If $B = e$, $f = !$ and B is of the form $\forall x D(x)$, then

$$S_2 = \langle D(x), X, !, D(x), \sigma[x/c_i]\rangle,$$

  where c_i is the constant chosen by *Challenger* (who here is Y) as a response to the attack ?-$\forall x/c_i$.

As usual, the notation '$\sigma[x/c_i]$' stands for the function that is otherwise like σ, but maps the variable x to c_i. Hence if σ is already defined on x, $\sigma[x/c_i]$ is the result of reinterpreting x by c_i; otherwise it is the result of extending σ by the pair (x, c_i).

- *Particle rule for existential quantifier:* If $B = e$, $f = !$ and B is of the form $\exists x D(x)$, then

$$S_2 = \langle D(x), X, !, D(x), \sigma[x/c_i]\rangle,$$

  where c_i is the constant chosen by *Defender* (who is X here), reacting to the attack ?-$\exists x$ of *Challenger* (that is, Y).

2.1.3 Structural rules

As we analyze dialogues, we will make use of the following notions: *dialogue*, *dialogical game*, and *play* of a dialogue. It is very important to keep them conceptually distinct. Dialogical games are sequences of dialogically signed expressions, i.e. expressions of the language $\mathcal{L}[\tau]$ equipped with a pair of labels, **P**-!, **O**-!, **P**-?, or **O**-?. The labels carry information about how the dialogue proceeds. Dialogical games are a special case of plays: all dialogical games are plays, but not all plays are dialogical games. However, all plays *are* sequences of dialogical games. Finally, dialogues are simply sets of plays.

A complete dialogue is determined by game rules. They specify how dialogical games in particular, and plays of dialogues in general, are generated from the thesis of the dialogue. Particle rules are among the game rules, but in addition to them there are so-called *structural rules*, which serve to specify the general organization of the dialogue.

Different types of dialogues have different kinds of structural rules. *When the issue is to test validity* − as it is for the dialogues considered in the present paper − a dialogue can be thought of as a tree, whose (maximal) branches are (finished) plays relevant for establishing the validity of the thesis. The structural rules will be chosen so that *Proponent* succeeds in defending the thesis against all allowed critique of *Opponent* if, and only if, the thesis is valid in the standard sense of the term ('true in every model'). In dialogical logic the existence of such a winning strategy for *Proponent* is typically taken as the *definition* of validity; however, this dialogical definition indeed captures the standard notion (cf. [Stegmüller 1964]; [Felscher 1985]; [Rahman 1993], I.4, Strategical Forms for First-Order Logic (Strategische Formen für Quantorenlogik), 88-107).

Each split into two branches − into two plays − in a dialogue tree should be considered as the outcome of a propositional choice made by *Opponent*. Any choice by **O** in defending a disjunction, attacking a conjunction, and reacting to an attack against a conditional, gives rise to a new branch: a new play. By contrast, *Proponent*'s choices do not generate new plays; and neither do *Opponent*'s choices for quantifiers (defending an existential quantifier, attacking a universal quantifier).

The participants **P** and **O** of the dialogues that we are here interested in − the dialogues used for characterizing validity − are of course idealized agents. If real-life agents took their place, it might happen that one of the players was cognitively restricted to the point of following a strategy which would make him lose against some, or even every sequence of moves by the Opponent − even if a winning strategy would be available to him. The idealized agents of the dialogues are not hence restricted: their 'having a strategy' means simply that there exists, by combinatorial criteria, a certain kind of function; it does not mean that the agent possesses a strategy in any cognitive sense.

Plays of a dialogue are sequences of dialogically signed expressions, and they share their first member, the *thesis* of the dialogue. In particular, plays can always be analyzed into dialogical games: any play is of the form $(\Delta_1, \ldots, \Delta_n)$, where the Δ_i are dialogical games $(i := 1, \ldots, n)$. The members of plays other than the thesis are termed *moves.* A move is either an attack or a defense. The particle rules stipulate exactly which moves are to be counted as attacks. Exactly those moves X-f-e whose expression component e is a first-order formula, are said to have *propositional content.* Recall that in the case of implication and negation some moves with propositional content count as attacks. (In the actual design of a dialogue there is usually a notational device to differentiate between those moves with propositional content that are attacks and those that are not.)

We move on to introduce a number of structural rules for dialogues designed for the language $\mathcal{L}[\tau]$. We will write $\mathcal{D}(A)$ for the dialogue about A, i.e. the dialogue whose thesis is A. Further, we will write $\Delta[n]$ for the member of the sequence Δ with the position n. Let A be a first-order sentence of vocabulary τ. We have the following structural rules (SR-0) to (SR-6) regulating plays $\Delta \in \mathcal{D}(A)$, i.e. members of the dialogue $\mathcal{D}(A)$.

(SR-0) (Starting rule):

(**a**) The dialogically signed expression $\langle \mathbf{P}\text{-!-}A \rangle$ belongs to the dialogue $\mathcal{D}(A)$: the thesis A stated by *Proponent* is itself a play in the dialogue about A.

(**b**) If Δ is any play in the dialogue $\mathcal{D}(A)$, then the thesis A has position 0 in Δ: If $\Delta \in \mathcal{D}(A)$, then

$$\Delta[0] = \langle \mathbf{P}\text{-!-}A \rangle.$$

(**c**) At even positions **P** makes a move, and at odd positions it is **O** who moves. That is, each $\Delta[2n]$ is of the form $\langle \mathbf{P}\text{-}f\text{-}B \rangle$ for some $f \in \{?, !\}$ and $B \in \mathit{Sub}(A)$; and each $\Delta[2n + 1]$ is similarly of the form $\langle \mathbf{O}\text{-}f\text{-}B \rangle$. Every move after $\Delta[0]$ is a reaction to an earlier move made by the other player, and is subject to the particle rules and the other structural rules.

(SR-1.I) (Intuitionistic round closing rule): Whenever player X has a turn to move, he may attack any (complex) formula asserted by his Opponent, Y, or he may defend himself against the *last not already defended attack* (i.e. the attack by Y with the greatest associated natural number such that X has not yet responded to that attack).

A player may postpone defending himself as long as he can perform attacks. Only the *latest* attack that has not yet received a response may be answered: If it is X's turn to move at position n, and positions l and m both involve an unanswered attack ($l < m < n$), then player X may *not* at position n defend himself against the attack of position l.

(SR-1.C) (Classical round closing rule): Whenever player X has turn to move, he may attack any (complex) formula asserted by his Opponent, Y, or he may defend himself against *any* attack, including those which have already been defended. That is, here even redoing earlier defenses is allowed.

(SR-2) (Branching rule for plays): If in a play $\Delta \in \mathcal{D}(A)$ it is **O**'s turn to make a propositional choice, that is, to defend a disjunction, attack a conjunction, or react to an attack against a conditional, then Δ extends into two plays $\Delta_1, \Delta_2 \in \mathcal{D}(A)$,[3]

$$\Delta_1 = \Delta^\frown\alpha \quad \text{and} \quad \Delta_2 = \Delta^\frown\beta,$$

differing in the chosen disjunct, conjunct *resp.* reaction, α *vs.* β. More precisely: Let $n \leq max\{m : \Delta[m]\}$.

- If $\Delta[n] = \langle \mathbf{O}\text{-!-}B \vee C \rangle$ and $\Delta[max] = \langle \mathbf{P}\text{-?-}\vee \rangle$, then

$$\alpha := \langle \mathbf{O}\text{-!-}B \rangle \quad \text{and} \quad \beta := \langle \mathbf{O}\text{-!-}C \rangle.$$

- If $\Delta[n] = \Delta[max] = \langle \mathbf{P}\text{-!-}B \wedge C \rangle$, then

[3] If $\bar{s} = (a_0, \ldots, a_n)$ is a finite sequence and a_{n+1} is an object, $\bar{s}^\frown a_{n+1}$ is by definition the sequence $(a_0, \ldots, a_n, a_{n+1})$. Generally, if $\bar{s} = (a_0, \ldots, a_n)$ and $\bar{s}' = (a'_0, \ldots, a'_{n'})$, then $\bar{s}^\frown\bar{s}' := (a_0, \ldots, a_n, a'_0, \ldots, a'_{n'})$. If $\bar{s} = \bar{s}_1{}^\frown\bar{s}_2$, then $\bar{s}_1$ is said to be an *initial segment* of $\bar{s}$, and, if the sequence $\bar{s}_2$ is not empty, its *proper* initial segment.

$$\alpha := \langle \mathbf{O}\text{-?-}L\rangle \quad \text{and} \quad \beta := \langle \mathbf{O}\text{-?-}R\rangle.$$

- If $\Delta[n] = \langle \mathbf{O}\text{-!-}B \to C\rangle$ and $\Delta[max] = \langle \mathbf{P}\text{-!-}B\rangle$, then

$$\alpha := \langle \mathbf{O}\text{ -?-}*\rangle \quad \text{and} \quad \beta := \langle \mathbf{O}\text{-!-}C\rangle,$$

  where $*$ is an attack marker corresponding to the logical form of B.

No moves other than propositional moves made by **O** will trigger branching.

(SR-3) (Shifting rule): When playing a dialogue $\mathcal{D}(A)$, **O** is allowed to switch between 'alternative' plays $\Delta, \Delta' \in \mathcal{D}(A)$. More exactly, if **O** loses a play Δ, and Δ involves a propositional choice made by **O**, then **O** is allowed to continue by switching to another play − existing by the *Branching rule* (SR-2). Concretely this means that the sequence $\Delta^\frown\Delta'$ will, then, be a play, i.e. an element of $\mathcal{D}(A)$.

It is precisely the *Shifting rule* that introduces plays which are not plain dialogical games. (Dialogical games are a special case of plays: they are identified with unit sequences of dialogical games.) As an example of applying the *Shifting rule*, consider a dialogue $\mathcal{D}(A)$ proceeding from the hypotheses $B, \neg C$, with the thesis $A := B \wedge C$. If **O** decides to attack the left conjunct, the result will be the play

$$(\langle \mathbf{P}\text{-!-}B \wedge C\rangle, \langle \mathbf{O}\text{-?-}L\rangle, \langle \mathbf{P}\text{-!-}B\rangle),$$

and **O** will lose. But then, by the *Shifting rule*, **O** may decide to do have another try. This time he wishes to choose the right conjunct. The result is the play

$$(\langle \mathbf{P}\text{-!-}B \wedge C\rangle, \langle \mathbf{O}\text{-?-}L\rangle, \langle \mathbf{P}\text{-!-}B\rangle, \langle \mathbf{P}\text{-!-}B \wedge C\rangle, \langle \mathbf{O}\text{-?-}R\rangle, \langle \mathbf{P}\text{-!-}C\rangle).$$

Observe that this play consists of two dialogical games, namely $(\langle \mathbf{P}\text{-!-}B \wedge C\rangle, \langle \mathbf{O}\text{-?-}L\rangle, \langle \mathbf{P}\text{-!-}B\rangle)$ and $(\langle \mathbf{P}\text{-!-}B \wedge C\rangle, \langle \mathbf{O}\text{-?-}R\rangle, \langle \mathbf{P}\text{-!-}C\rangle)$. By contrast, it is not itself a dialogical game.

(SR-4) (Winning rule for plays): A play $\Delta \in \mathcal{D}(A)$ is *closed*, if $\Delta = (\Delta_1, \ldots, \Delta_n)$, where the Δ_i are dialogical games, and in the most recent dialogical game Δ_n there appears the same positive literal in two positions, one stated by X and the other one by Y. That is, Δ is closed if for some $k, m \in \mathbb{N}$ and some positive literal $\ell \in Sub(A) \cup \{A\}$, we have:

$$\Delta_n[k] = \ell = \Delta_n[m],$$

where $k < m$ and furthermore, k is odd if, and only if m is even or equal to zero. If this condition is not satisfied, Δ is *open*.

If a play is closed, the player who stated the thesis (that is, **P**) *wins* the play; otherwise he loses it. A play is *finished*, if it is either closed, or else such that no further move is allowed by the particle rules or (other) structural rules. If a play is finished and open, **O** wins the play. Observe that whenever a play $\Delta \in \mathcal{D}(A)$ is finished, there is no further play $\Delta' \in \mathcal{D}(A)$ such that Δ is an initial segment of Δ'.

(SR-5) (Formal use of atomic formulas): **P** cannot introduce positive literals: any positive literal must be stated by **O** first. Positive literals cannot be attacked.

In the following we will consider, when speaking of First-order logic, intuitionistic dialogues with additional hypotheses of the following form:

$$\forall x_1 \ldots \forall x_n(\mathrm{R}x_1 \ldots x_n \vee \neg \mathrm{R}x_1 \ldots x_n),$$

where R is a relation symbol of a fixed vocabulary τ. That is, the relevant hypotheses are instances of (a universal closure of) *tertium non datur*. In the presence of such hypotheses, we may use a more general formulation of the rule (SR-5):

(SR-5*): **P** cannot introduce literals: any literal (positive or not) must be stated by **O** first. Positive literals cannot be attacked.

Before we can state the structural rule (SR-6), or the *'No delaying tactics' rule*, we need some definitions.

Definition 2.2 (Strict repetition of an attack *resp.* a defense)

(a) *We speak of a* strict repetition of an attack*, if a move is being attacked although the same move has already been challenged with the same attack before. (Notice that even though choosing the same constant is a strict repetition, the choice of ?-L and ?-R are in this context different attacks.)*
In the case of moves where a universal quantifier has been attacked with a new constant, the following type of move must be added to the list of strict repetitions:

> *A universal quantifier move is being attacked using a new constant, although the same move has already been attacked before with a constant that was new at the time of that attack.*

(b) *We speak of a* strict repetition of a defense*, if a challenging move (attack) m_1, which has already been defended with the defensive move (defense) m_2 before, is being defended against the challenge m_1 once more with the same defensive move. (Notice that the left part and the right part of a disjunction are in this context two different defenses.)*
In the case of moves where an existential quantifier has been defended with a new constant, the following type of move must be added to the list of strict repetitions:

> *An attack on an existential quantifier is being defended using a new constant, although the same quantifier has already been defended before with a constant that was new at the time of that defense.*

Notice that according to these definitions, neither a new defense of an existential quantifier, nor a new attack on a universal quantifier, represents a strict repetition, if *it uses a constant that is not new but is however different from the one used in the first defense (or in the first attack).*

(SR-6) ('No delaying tactics' rule): This rule has two variants, classical and intuitionistic, depending on whether the dialogue is played with the classical structural rule (SR-1.C), or with the intuitionistic structural rule (SR-1.I).

Classical: No *strict* repetitions are allowed.

Intuitionistic: If **O** has introduced a new atomic formula which can now be used by **P**, then **P** may perform a repetition of an attack. No other *strict* repetitions are allowed.

Definition 2.3 (Validity) *A first-order sentence A is said to be* dialogically valid *in the classical (intuitionistic) sense, if all plays belonging to the classical (*resp. *intuitionistic) dialogue $\mathcal{D}(A)$ are closed.*

As was pointed out already above, it is possible to prove that a first-order sentence is valid (in the usual model theoretical sense) if, and only if, it is dialogically valid (i.e. valid in the sense of Definition 2.3).

Let us take two examples of dialogues, one classical and the other intuitionistic.

Example 2.4 *Consider the classical dialogue $\mathcal{D}(p \vee \neg p)$. Its thesis is $p \vee \neg p$, where p is an atomic sentence. In Figure 1, a dialogical game from dialogue $\mathcal{D}(p \vee \neg p)$ is described. This dialogical game is won by* **P***:*

	O		**P**	
			$p \vee \neg p$	0
1	?-$\vee$	0	$\neg p$	2
3	p	2	–	
[1]	[?-$\vee$]	[0]	p	4

Figure 1. Classical rules, **P** *wins.*

The outer columns indicate the position of the move inside the dialogical game, while the inner columns state the position of the earlier move that is being attacked. The defense is written on the same line with the corresponding attack: an attack together with the corresponding defense constitutes a so-called closed round*. The sign "–" indicates that there is no possible defense against an attack on a negation.*

In the dialogical game of the example, **P** *wins because after* **O***'s last attack in move 3,* **P** *is allowed – according to the classical rule SR-1.C – to defend (once more) himself against* **O***'s attack made in move 1, and so the game in question is closed.* **P** *states his new defense in move 4. (In reality* **O** *does not repeat his attack of move 1: what we have written between square brackets simply serves to remind of the attack against which* **P** *is re-acting.)*

In fact the described dialogical game is the only finished play of the dialogue $\mathcal{D}(p \vee \neg p)$: **O** *could not prolong the play any further by making different moves. Hence not only does* **P** *win the described particular dialogical game – in fact he has a winning strategy in the dialogue, i.e. he is able to win no matter what* **O** *does. In other words, the sentence $p \vee \neg p$ is dialogically valid in the classical sense (cf. Definition 2.3).* ■

Example 2.5 *Let us consider the intuitionistic variant of the dialogue of the above example. In Figure 2, a dialogical game from the intuitionistic dialogue $\mathcal{D}(p \vee \neg p)$ is described. This game is won by* **O***:*

	O		**P**	
			$p \vee \neg p$	0
1	?-$\vee$	0	$\neg p$	2
3	p	2	–	

Figure 2. Intuitionistic rules, **O** *wins.*

It is **O** *who wins the dialogical game of the example: the game is open, and no further move is possible following the intuitionistic structural rules. In particular remaking an earlier move – as in the above example of a classical dialogue – is not possible.*

In fact **O** *has trivially a winning strategy in the intuitionistic dialogue* $\mathcal{D}(p \vee \neg p)$*:* **P** *cannot prevent, by making different moves,* **O** *from generating precisely the described play won by* **O**. *Observe, in particular, that the sentence* $p \vee \neg p$ *is* not *dialogically valid in the intuitionistic sense. (This does not mean, of course, that thereby the sentence* $\neg(p \vee \neg p)$ *would be intuitionistically valid!)* ∎

2.2 Material dialogues and truth in a model

Let us recall here how do dialogicians tackle the question of characterizing truth of a sentence (satisfiability of a formula) relative to a model.

There are two rather straightforward approaches one can assume; they give rise to what are known as 'alethic' and 'material' dialogues (see e.g. [Rahman & Keiff 2005]. As dialogues are designed for dealing with validity, some additional ingredient must be introduced into dialogues in order to make them capable of dealing with material truth. *Alethic dialogues* are simply obtained by relativizing a dialogue to a model. Hence a part of the specification of an alethic dialogue in the case of Propositional logic will be a valuation function, and in the case of First-order logic a τ-structure for an appropriate vocabulary τ.

By contrast, the idea behind *material dialogues* is to avoid having an extra component to dialogues (such as a specification of a model); they are meant to do with the resources of dialogues proper (which are designed for dealing with validity), and the idea is to 'approximate' characterization of truth by adding a sufficient amount of *additional hypotheses* − taken to be *initial concessions* of *Opponent* − which would serve to specify a model by using the resources of the object language only.

What is a sufficient amount, then? In the case of Propositional logic, when discussing the truth of a formula A, any relevant model can indeed be specified in terms of **PL**-formulas, namely literals: positive or negative atomic formulas. What is more, it suffices to specify a *finite* number of such literals. The relevant models are identified by going through all propositional atoms p_i *appearing in* A (there are only finitely many of these atoms) and choosing, for all i, either p_i itself or its negation $\neg p_i$. In this way any relevant model − any truth-value distribution − can be specified.

For First-order logic, this approach has the obvious downside that in general there is no way of capturing a τ-structure in terms of a finite number of first-order sentences. Take for example a $\{\mathrm{P}\}$-structure $\mathcal{M}$ with an infinite domain $\mathrm{M} = \{d_i : i < \omega\}$, where P is unary. To exhaustively describe $\mathcal{M}$ in terms of first-order sentences, we need an infinite list $\langle \ell_i : i < \omega \rangle$ of literals, where $\ell_i := \mathrm{P}c_i$, if $d_i \in \mathrm{P}^{\mathcal{M}}$, and $\ell_i := \neg\mathrm{P}c_i$, if $d_i \notin \mathrm{P}^{\mathcal{M}}$. By stipulation, the constant c_i stands for the element d_i.

Mathematically there is of course nothing problematic with such infinite lists of hypotheses. But one *desideratum* in designing dialogues typically is that it should be possible to think of them as humanly manageable, ideally temporal processes. Such a process cannot really begin by going through an infinity of hypotheses. This is why material dialogues with an infinity of hypotheses should be considered as something deeply unsatisfactory. So what can we do? A solution to this dilemma, found by Tero Tulenheimo, has been developed in [Rahman & Tulenheimo 2005]: Given a vocabulary of one unary relation symbol P, a relevant piece of finite information that the models of classical First-order logic must satisfy, is expressed by this sentence:

$$\forall x(\mathrm{P}x \vee \neg \mathrm{P}x).$$

That is, whatever is the value c of x, the instance $\mathrm{P}c \vee \neg \mathrm{P}c$ of *tertium non datur* must hold.

What type of question concerning the sentence $\forall x(\mathrm{P}x \vee \neg \mathrm{P}x)$ should we make in order for the answer to identify a (possibly infinite) model? Clearly, we should ask, once and for all, *Opponent* to choose a *Skolem function* for this sentence. No finite amount of questions of the type ?-$\forall x/c$ will reveal this information, if there are infinitely many values for x available. On the other hand, a Skolem function

$$f : \{c_0, c_1, \ldots\} \rightarrow \{\texttt{left}, \texttt{right}\}$$

expressly says, for each possible value c_i of x, which of the disjuncts *Opponent* considers being satisfied by c_i. This is exactly what it means to specify a model of vocabulary $\{\mathrm{P}\}$ with the domain $\{c_0, c_1, \ldots\}$.

Now for a finite vocabulary τ, the finite information we use for specifying a model will be extracted from the sentences

$$\forall x_1 \ldots \forall x_n(\mathrm{R}x_1, \ldots, x_n \vee \neg \mathrm{R}x_1, \ldots, x_n),$$

one for each $\mathrm{R} \in \tau$ of arity n. We introduce a new mode of question, which enables to ask about a Skolem function for an operator.[4] When asked about a sentence of the form $\forall \bar{x}(R\bar{x} \vee \neg R\bar{x})$, the question

$$??\text{-}\vee$$

must be answered by providing a second-order object, namely a Skolem function

$$f : \{c_i : i < \omega\}^n \rightarrow \{\texttt{left}, \texttt{right}\}$$

for the unique token of the disjunction $\vee$ appearing in $\forall \bar{x}(R\bar{x} \vee \neg R\bar{x})$. (the double '?' indicates that the answer should be a second-order object.) If **O** asserts that f is such a function, **P** is able to read the interpretation of the relation symbol R from the function f. **P** can draw all kinds of inferences from it, for instance check whether the relation symbol is satisfied by at least one tuple, by checking whether $\texttt{left} \in Im(f)$ or not.

The way in which we extract a model from the sentences $\forall \bar{x}(R\bar{x} \vee \neg R\bar{x})$ is by taking them as initial concessions of **O**, and asking the question of ??-$\vee$ with respect to each sentence. The questions ??-$\vee$ give rise to the following new structural rule:

(SR-7) (Skolem function rule): If **O** has conceded that f is a Skolem function for $\vee$ in

$$\forall \bar{x}(\mathrm{R}\bar{x} \vee \neg \mathrm{R}\bar{x}),$$

then **O** must also, if asked, concede all instances of this second-order concession. That is, for any tuple $\bar{c}$ interpreting the variables $\bar{x}$, he must concede $\mathrm{R}\bar{c}$, if $f(\bar{c}) = \texttt{left}$, and $\neg \mathrm{R}\bar{c}$, if $f(\bar{c}) = \texttt{right}$. Accordingly, after **O** has replied by some f to a question ??-$\vee$, **P** is always entitled to pose the question

$$?\text{-}f/\bar{c},$$

[4] The barred x, i.e. $\bar{x}$, stands for a finite sequence of variables, $x_1 \ldots x_n$, and $\forall \bar{x}$ stands for $\forall x_1 \ldots \forall x_n$.

for any tuple $\bar{c}$, asking **O** to confirm that indeed he concedes that the tuple $\bar{c}$ satisfies the disjunct $f(\bar{c})$. **O** has no real choice for his answer: the reply is fully predetermined by his choice of f and the requirement that **O** must be coherent in his replies.

In the present paper we will assume this result but we will use instead a simplified version of material dialogues inspired in [Hintikka et al., 1996]'s interrogative inquiry or deductive-interrogative games. We will allow to formulate questions in relation to the truth of relevant atomic formulae in the model - Hintikka calls these type of moves *interrogative moves* or *questions to the oracle* -. These answers will appear in the object language as explicit concessions of the Opponent. We used "oracle consultation" moves to perform this bring the relevant aspects of the model into the object language. Indeed, the main characteristic, following [Hintikka et al., 1996], of an oracle is that :

> All of the oracle's answers are true, and known by the inquirer [Proponent] to be true.

and the main restriction is that :

> all available answers are quantifier-free, i.e. particular propositions. In view of the Yes-No Theorem this case is to all practical purposes the one in which we restrict our attention to questions of the form "A or not-A?" where A is an atomic proposition.

The Yes-No Theorem teaches us precisely that any conclusion you might have been derived with logical and interrogative moves can be derived in the same logic by using only yes-no (Socratic) questions. This postulate has been

> tacitly assumed [as] applicable in empirical reasoning [...] the relevant [being] nature.

However, as already mentioned, there are no definite reasons for such an atomistic restriction and quite a lot speaks to allow Wh-questions such as:

> "Which individual, call it x, is such that S[x].)

But this is what the skolem-move mentioned above performs as developed in [Rahman & Tulenheimo 2005].

Actually if we want be sure that validity is also captured we will need a further and more precise elaboration of this step but this will not be tackled here but the general idea should be clear by now: classical validity is achieved when the dialogues include all the possibilities of building a model. This is the point of the formal rule in dialogic, which does not allow to challenge atomic formulae of the Opponent. In the our present context this amounts to eliminate the difference between oracle and Opponent moves.

3 IF-Logic

3.1 IF First-Order Syntax

In IF First-Order Logic quantifiers will have the forms

- $\forall x/y_1, \ldots, y_n$ and
- $\exists x/y_1, \ldots, y_n$

It is assumed that these quantifiers lie in the syntactic scope of quantifiers $Q_1y_1 \ldots Q_ny_n$ (with $Q_i \in \{\exists, \forall\}$)

Let us also add conjunctions and disjunctions that are independent of syntactically subordinate quantifiers in the same sense as described above

- $\wedge/y_1,\ldots,y_n$ and
- $\vee/y_1,\ldots,y_n$

Thus we assume here too that these connectives lie in the syntactic scope of quantifiers $Q_1y_1 \ldots Q_ny_n$ (with $Q_i \in \{\exists, \forall\}$)

4 IF Dialogues

4.1 The dialogic of imperfect transmission of information

The basic idea of these dialogues is simple and pretty straightforward:

While challenging a quantifier that is independent of at least one given logical constant a player may hide in a box the information corresponding to the value of the corresponding variable. This box can be opened and information made public if there is a variable dependent on the variable, which yielded the creation of the box. The opening of the box, might motivate to pose an *oracle-question* by the answer of which the Opponent will have to answer if the atomic formula at stake or its negation is true in the given model.

4.1.1 Extensions of the language

We extend the standard IF-syntax and semantics with the following devices: let $[i]$ be a *box*. The number i is a label indicating that the box $[i]$ was introduced at the i^{th} move. A box hides a constant c such that $c \in Const$.

Let I be a function that assigns to a box the constant it is hiding, $I : [i] \mapsto c$ Thus, we say that a box $[i]$ hides a constant c iff $c \in Const \wedge I([i]) = c$

Let *Boxes* be the set of boxes.

Definition 4.1 (Box-Formulae) *Let us define the set* $Box - Formulae = \{\phi : \phi = R_i(t_1, \ldots t_n)\}$ *such that for any* t_i *such that* $1 < i < n$ *with* $\nu(R_i) = n$ *we have* $t_i \in Terms \cup Boxes$ *and for some* t_i *such that* $1 < i < n$ *with* $\nu(R_i) = n$ *we have* $t_i \in Boxes$.

The rest of the definitions will be given, as usual in dialogic, with help of appropriate particle rules.

4.2 Particle rules

We will here slightly change the standard notation of non IF-dialogic in order to avoid confusion by the use of the slash which in standard particle rules is used to signalize substitution of variable

4.2.1 Producing boxes

- While challenging a universal quantifier $\forall x_i$ the challenger may keep his choice private by storing it a box if and only if the formula where $\forall x_i$ occurs contains at least one quantifier or propositional connective independent of precisely $\forall x_i$. In fact, the challenge consists in two moves:

  1. the public move, where a box is produced and
  2. the private move, where the content of the box is stored in a secret place

Assertion	X-!-$\forall x_1$ ...$Q_n(x_n/x_1)$ ...x_1...x_n with $Q_n \in \{\exists,\ \forall\}$
Attack	*public move:* Y-?-$\forall x_1\ := [i]$ *private move:* Y-$I([i]) := c_j$ for any c_j Y may choose
Defense	X-!-$Q_n(x_n/x_1)$ ...$[i]$...x_n

Assertion	X-!-$\forall x_1 \ldots \{\vee, \wedge\}/x_1\}\ldots$
Attack	*public move:* Y-?-$\forall x_1\ := [i]$ *private move:* Y-$I([i]) := c_j$ for any c_j Y may choose
Defense	X-!-$\ldots [i] \ldots \{\vee, \wedge\}/x_1\} \ldots [i] \ldots$

The expression W-$I([i]) := c_j$ signalizes that the player W hides the constant c_j in the box $[i]$. We assume that this move is stored in a secret place accessible only to the player that has hidden the constant, until the box is opened.

- The case of the existential quantifier is perfectly dual. Indeed, while defending an existential quantifier $\exists x_i$ the defender may hide his choice in a box and store this information in a secret place if and only if the formula where $\exists x_i$ occurs contains at least one quantifier or propositional connective independent precisely of $\exists x_i$. Here it is the defense, which splits into a public and a private move. We will not write down the formal presentation of the particle rule neither here not further on the paper if the formulation is just the dual of the case of the quantifier for which the precise particle rule has been written down.

4.2.2 Opening boxes

- A player may ask to open a box before defending an existential quantifier if this quantifier is dependent on another one and the latter has been already challenged with a box:

Assertion	X-!-$\exists x_n \ldots [1] \ldots x_n$
Attack	Y-?-$\exists x_n$
Defense	X-!-*Open*[1]

- A player may ask to open a box before challenging a universal quantifier under exact dual conditions to the case described above.

- Box-Formulae[5] can be challenged by opening the box and answered by replacing the box with the constant hidden in the box - we will say then that the content of the box has been now made public. In fact it could happen that the player who has hidden the box is the same player who later asks to make it public. In this case the move *open the box* includes the information of the content of the box:

Assertion	X-!-$\phi([1],[2],c_1)$
Attack	Y-*Open* [1]
Defense	X-!-$\phi(c_i,[2],c_1)$ with $c_i = I([1])$

[5]definition 4.1

In fact it could happen that the player X who has hidden the box is the same player who later asks to make it public. In this case it is indeed the player X who provides the content of the box with his attack *open the box* whereas the player Y must use this information to implement this information in the formula at stake:

Assertion	X-!-$\phi([1],[2],c_1)$
Attack	Y-*Open* [1] with $c_i = I([1])$
Defense	X-!-$\phi(c_i,[2],c_1)$

- Dummy quantifiers: Let us assume that there is a dummy quantifier $\exists z$ which occurs between two quantifiers $\forall x$ and $\exists y$ and let us further assume that $\exists y$ is independent of $\forall x$ but not of the dummy quantifier $\exists z$. In this case the attack on the dummy quantifier can be answered by replacing y with z. This move produces the fall of the slash and in the literature it is known as a case of *signaling* - the next move after this substitution will produce the opening of the box.

Assertion	X-!- ...[i]...$\exists z\ \exists y/x\ ...y_n$ with $x_i := ([i])$
Attack	Y-?-$\exists z$
Defense	X-!-$\ldots [i]...\exists z\ ...z_n$

4.2.3 Closing opened boxes

- A player may ask to close an opened box if that box has been opened and the player must now make a move in relation to a logical constant K_j which is independent of the variable that motivated the box:

Assertion	X-!-$\phi(c_i,\ldots,K_j/x_i)$ with $x_i := [1]$ and $c_i = I([1])$
Attack	Y-*Close*c_i
Defense	X-!-$\phi([1],\ldots,K_j/x_i)$

4.2.4 Independent logical constants

- Moves on logical constants containing slashes will follow the standard particle rules

4.2.5 Oracle moves

As already mentioned at the end of 2.2 we will assume the results on material dialogues obtained in [Rahman & Tulenheimo 2005], with the addition of *oracles-moves* which will allow to formulate questions in relation to the truth of relevant atomic formulae in the model. The answers will appear in the object language as explicit concessions of the Opponent in relation to truth of a given atomic formula or its negation.

The following rules for such oracle-moves are not really particle rules because they are not player independent. The reason is that in general, with the exception of atomic moves resulting from opening a box, because of the formal rule, we assume atomic moves of the Opponent as true.

- The move *open the box* may be counterattacked by appealing to a classical oracle about an atomic formulae or its negation, the oracle will answer without any delay through the Opponent and will yield a concession of either the positive or the negative literal:

Assertion	X-*Open* [1]
Counterattack	Y-?*Oracle*$(\phi, \neg\phi)$ with $\phi \in$ *Atoms*
Defense	X-!-ϕ resp. $\neg\phi$, for any of the literals true in the model

- Atomic formulae, which resulted as an answer of the Opponent to an open the box attack, may be challenged. The challenge consists here too in a classical oracle move:

Assertion	X-ϕ (resp. $\neg\phi$)
Attack	Y-?*Oracle*$(\phi, \neg\phi)$ with $\phi \in$ *Atoms*
Defense	X-!-ϕ resp. $\neg\phi$, for any of the literals true in the model

4.3 Structural rules

1. We will assume the intuitionistic structural rules
2. We will introduce the following exception to the formal rule: Atomic formulae of the Opponent can be challenged iff they result from opening a box.

4.4 Examples

Let us use the following notation for dialogues: From top to bottom the tableau for a dialogue has been divided in two sections:

1. The private section PS containing the stock of private information
2. The public section PUS containing the explicit information in the dialogue (the public part of the argumentation)

From left to right the columns of PS indicate the following :

- The first column of PS contains an expression such as $\xrightarrow{i}$ which indicates that at move i the Opponent packed a given constant into a box .
- The second column of PS contains an expression of the type $PS\ [i] := c_j$ indicating that the Opponent has hidden the constant c_j into the box $[i]$,
- The third column of PS is always empty because it displays the actual public moves of the dialogue.
- The fourth column of PS contains an expression such as $\overrightarrow{k}$ which indicates that the box at stake was opened at move k.
- the four next columns of PS display analogous private information of the Proponent.

From left to right the columns of PUS indicate the following :

- The first, third and fourth columns of PUS correspond to the usual Opponent's columns of the standard dialogues.
- The second column of tPUS is empty.
- the four next columns of PUS display analogous public information of the Proponent.

Stipulation 4.2

- In all our examples we will assume that the interpretation $I(c_i)$ of c_i is δ_i, where $\delta_i \in Dom$, and Dom is the domain of the model given in each example.
- In all our examples we use the relation of identity $=$ defined as usual with $=$: $\{< \delta_i, \delta_i >$.

Example 4.3 *Consider a dialogue for the IF-sentence :*

$$\forall x \; \exists y/x \; \neg(x = y) \tag{1}$$

The domain is $\{\delta_1, \delta_2\}$.
Let us play:

Os				***Ps***			
$\overset{1}{\rightarrow}$	$[1] := c_1$		$\overset{\rightarrow}{6}$				
O				***P***			
					$\forall x \; \exists y/x \; \neg x = y$		0
1		$?x := [1]$	0		$\exists y/x \; \neg[1] = y$		2
3		$?\exists$	2		$\neg[1] = c_1$		4
5		$[1] = c_1$	4		$-$		
7		$c_1 = c_1$		5	$open[1]$		6
9		$c_1 = c_1$		7	$?oracle(c_1 = c_1, \neg c_1 = c_1)$		8

P *loses.*

Claim 4.4 ***P*** *loses in this play, however* ***O*** *does not have a winning strategy.*

Proof.

Os				***Ps***			
$\overset{1}{\rightarrow}$	$[1] := c_1$		$\overset{\rightarrow}{6}$				
O				***P***			
					$\forall x \; \exists y/x \; \neg x = y$		0
1		$?x := [1]$	0		$\exists y/x \; \neg[1] = y$		2
3		$?\exists$	2		$\neg[1] = c_2$		4
5		$[1] = c_2$	4		$-$		
7		$c_1 = c_2$		5	$open[1]$		6
9		$\neg c_1 = c_2$			$?oracle(c_1 = c_2, \neg c_1 = c_2)$		8
		$-$			$c_1 = c_2$		10

P *wins.*

■

Example 4.5 (Signaling) *Consider the dialogue for the IF-sentence :*

$$\forall x\ \exists z\ \exists y/x\ \neg(x = y) \tag{2}$$

The domain is $\{\delta_1, \ldots \delta_i, \ldots \delta_n\ \}$ *with* $\{1, \ldots i, \ldots n\} \subseteq \mathbb{N}^+$.

In the play for $\mathcal{D}(\forall x\ \exists z\ \exists y/x\ \neg(x = y))$ *P wins using the move for the dummy quantifier:*

1. *The Opponent will start by hiding the value of* x *in a box and proceeds by attacking the dummy quantifier.*
2. *The Proponent can then (because of the dummy-quantifier move) replace* y *with* z.
3. *The Opponent will the proceed to attack the reformulated quantifier.*
4. *The Proponent can then ask to open the box, consult the oracle, use the corresponding information, and win.*

We leave it to the reader to work out the graphical details.

Example 4.6 (Opening and Closing boxes) *Let us run a dialogue for the well known IF-example :* $\forall x\ \exists y\ \forall z/x, y\ \exists w/x, y\ R(x, y, z, w)$ *is a wff of IF First-Order dialogical logic.*

Let us assume an appropriate model

1. *The Opponent will start by hiding the value of* x *in a box and proceeds by attacking* $\exists y$.
2. *The Proponent can, before answering (because of the dependency), ask to open the box for the value of* x *and respond then accordingly.*
3. *The Opponent can now, before challenging* $\forall z$ *(because of the independence), ask to close the boxes for the values of* x *and* y *and then proceed with the attack on the quantifier binding* z. *After the Proponent's response to the latter challenge, the Opponent will challenge* $\exists w$.
4. *The Proponent can, before answering (because of the dependency), ask to open the box for the value of* z, *consult the oracle and respond then accordingly to the model.*

Example 4.7 (Proponent's Independences) . *Let us discuss the dialogue for*

$\forall y\ \exists z\ \exists v/y, z\ (Rc_1y\ \rightarrow\ (Ryz\ \wedge\ Rzv))$

This example provided by Tero Tulenheimo relates to a series of cases where it makes sense to talk of independences between Proponent's own moves. In fact this represents a main difference with the poker-dialogues of the next section (see discussion of the last example of the paper).

The domain of our model contains the following objects:

$\{\delta_1, \delta_{2.1}, \delta_{2.2}, \delta_{3.1}, \delta_{3.2}, \delta_{3.3}, \delta_{3.4}, \delta_{4.1}, \delta_{4.2}, \delta_{43}\}$

The notation for the elements of the domain of the model might look idiosyncratic but actually the interpretation of R in our model has been built up on this notation. Thus $\delta_{j.n(j=i+1)}$ is an object such that $R < \delta_i, \delta_{i+1.n} >$ holds. Here all the details of the interpretation of R:

$\{< \delta_1, \delta_{2.1} >, < \delta_1, \delta_{2.2} >, < \delta_{2.1}, \delta_{3.1} >, < \delta_{2.1}, \delta_{3.2} >, < \delta_{2.2}, \delta_{3.3} >, < \delta_{2.2}, \delta_{3.4} >, < \delta_{3.1}, \delta_{4.1} >, < \delta_{3.2}, \delta_{4.2} >, < \delta_{3.3}, \delta_{4.2} >, < \delta_{3.4}, \delta_{4.3} >\}$.

We assume that δ_1 is the interpretation of the constant c_1 and that the interpretation of $c_{i.n}$ is $\delta_{i.n}$.

The reader can verify that despite the independences Proponent wins the dialogue. Indeed, if the Opponent chooses on of the values $c_{2.1}$ or $c_{2.2}$ for y which verify the antecedent of the conditional (if the Opponent chooses one value for y which does not verify the antecedent the Proponent wins anyway), then the Proponent can win choosing the appropriate value for z and then always choose $c_{4.2}$ as value for v. In fact the formula is equivalent to the formula:

$\exists v\ \forall y\ \exists z\ (Rc_1y \rightarrow (Ryz \wedge Rzv))$ *and not to*

$\forall y\ \exists z\ \exists v\ (Rc_1y \rightarrow (Ryz \wedge Rzv))$

4.5 Poker-Dialogues: IF-dialogic with information delay

Let us assume a game where an action has to be chosen by a player X under conditions of information gaps. This general situation can be taken as describing a case of imperfect information. Namely, as a game where player X does not know what actions have been previously chosen (by the other player or even by X himself. Accordingly, this situation will be taken as conveying a case where the information in relation to the relevant choices has somehow not been perfectly transmitted. Now such a general situation of information gaps can be taken instead as describing a case of information delay. In the latter interpretation the information gap is understood as displaying a situation where, when choosing takes place, some relevant information is not given yet (cf. e.g. [Kushmerick et al., 1995]). Take as an example poker (Hold'em) where at each hand two cards are distributed, the precise value of which are private to the player who received those cards. Here it might be possible to choose depending not upon the concrete values of the cards of the other player but as function of some information gathered during the game. Now, certainly, every player has knowledge of the value he has chosen, therefore when a player gives an algorithm of decision making depending on a value he has chosen, all he considers is the value he should choose, knowing the previously chosen value. You cannot choose independently of a value you have chosen. In poker playing independently of your knowledge of the card, is still a way to play dependently. The equivalent of not watching your cards in poker is not possible in a game when you also choose the value of your cards, because trivially you have to choose their value.

Analogously, in poker dialogues a player might have to play being uncertain of some information until the end of the play. One interesting point is that poker dialogues, which are a kind of games under uncertainty might be thought as being situated between imperfect and incomplete information games for logic. Indeed as shown by Ahti Pietarinen ([Pietarinen 2005]) incomplete information games, usually the game-theoretical frame for modeling uncertainty, can be implemented in logic by IF-games where there is a third player called *Nature* (equivalent to Hintikka's *oracle*) who could hide information to both Eloise and Abelard. In the case of Poker dialogues this could mean that the oracle lies or makes mistakes.

Stipulation 4.8 (Characterizing truth)

- A *sentence will be true in the model if the Proponent's strategy is a winning strategy, i.e. a strategy such that when the play is repeated the Opponent does not win although the Opponent was allowed to make a new choice and the Proponent was forced to keep the same strategy and the Opponent was aware of this.*
- *Dually, a sentence will be false in the model if the Opponent's strategy is a winning strategy.*
- *If the players who win both plays are not the same the truth value of the sentence will be undefined in the model.*

4.5.1 Extensions of the language

We will need to extend further on the language of 4.1.1

Definition 4.9 (Function of boxes) *Let $f(X)$ be a function with domain Boxes and with range Const.*

Let $f_\vee(X)$ be a function with domain Boxes and the range of which are the two correspondent proper subformulae.

Let $f_\wedge(X)$ be a function with domain Boxes and the range of which is $\{?L, ?R\}$.

4.6 Particle rules

4.6.1 Producing boxes

- The same as in 4.2

4.6.2 Dependent quantifiers

- An existential quantifier *dependent* of at least one quantifier that has been defended or challenged with boxes (functions), may be defended with a function of any or all of those boxes (functions).

Assertion	X-!-$\exists x_n/x_1\ \psi([1],[2],x_n)$ with $:= (x_1) = [1]$ and $:= (x_2) = [2]$
Attack	Y-?-$\exists x_n$
Defense	X-!-$\psi([1],[2],f([2]))$, X-$x_n := f([2])$

- We leave it to the reader to produce the table for the case of functions of functions.
- The exact dual holds for the universal quantifier.

4.6.3 Independent logical constants

- Same as in 4.2

4.6.4 Opening the boxes

Box-Formulae[6] can be challenged by opening the box and revealing to both players the hidden value and answered by replacing the box with the constant hidden in the box:

Assertion	X-!-$\phi([1],[2],c_1)$
Attack	Y-*Open* [1]
Defense	X-!-$\phi(c_i,[2],c_1)$ with $c_i = I([1])$

4.6.5 Function of boxes

Formulae containing a function f of boxes, can be challenged by asking for the value of the function f of these boxes and answered by replacing the function of the boxes with the correspondent value.

If it is the challenger who has introduced the f function, he also defines the value of the function.

Assertion	X-!-$\phi x_i(f([1],[2]),c_1)$
Attack	Y-?$f([1],[2])$, Y-$f([1],[2]) := ([1]*[2])$
Defense	X-!-$\phi x_i([1]*[2]),c_1$

Else the defender defines the value of the function f.

Assertion	X-!-$\phi_i(f([1],[2]),c_1)$
Attack	Y-?$f([1],[2])$
Defense	X-!-$\phi_i([1]*[2]),c_1$, X-$f([1],[2]) := ([1]*[2])$

$([1]*[2])$ being the result any finite number of successive arithmetical functions which yielding appropriate constants.

4.6.6 Independent connectives

Challenging a conjunction or defending a disjunction which is *independent* of all other connectives or quantifiers follows the standard particle rules for FOL.

4.6.7 Dependent connectives

A conjunction the evaluation of which is *dependent* of at least one quantifier that has(have) been challenged with a box (with boxes) may be challenged with a $f_\wedge$ function, that is a function of that(these) box(es) with range $\{?L, ?R\}$.

---

[6]definition 4.1

Assertion	X-!-$\psi([1], c_1) \wedge \phi([1], c_1)$
Attack	Y-$f_\wedge([1])$
Defense	if $Y - f_\wedge([1])$:=?L, then X-!-$\psi([1], c_1)$ else if $Y - f_\wedge([1])$:=?R X-!-$\phi([1], c_1)$

A $\phi \vee \psi$-disjunction the evaluation of which is *dependent* of a (resp. several) quantifier(s) that has(have) been challenged with a box (with boxes) may be challenged with a $f^\vee$ function, that is a function of that(these) box(es) with range $\{\phi, \psi\}$.

Assertion	X-!-$\psi([1], c_1) \vee \phi([1], c_1)$
Attack	Y-?$\vee$
Defense	X-!-$f_\vee([1])$

4.6.8 $f_\vee([1])$-function and $f_\wedge([1])$-function

$f_\vee([1])$-function and $f_\wedge([1])$-function may be challenged by asking for the value of the function.

Assertion	X-$f_\vee([1])$
Attack	Y-?$f_\vee([1])$
Defense	X-!-ψ or ϕ, X chooses

Assertion	X-$f_\wedge([1])$
Attack	Y-?$f_\wedge([1])$
Defense	X-?L or X-?R, X chooses

An attack on a $f_\vee([1])$-function and $f_\wedge([1])$-function may be challenged by asking to the other player to assert either the positive or the negative boxatomic formula at stake.

Assertion	X-?$f_\vee([1])$
Attack	Y-?$(\phi[1], \neg\phi[1])$ with $\phi[1] \in Box - formulae$
Defense	X-!-$\phi[1]$ resp. $\neg\phi[1]$, X chooses

Assertion	X-?$f_\wedge([1])$
Attack	Y-?$(\phi[1], \neg\phi[1])$ with $\phi[1] \in Box - formulae$
Defense	X-!-$\phi[1]$ resp. $\neg\phi[1]$, X chooses

4.6.9 Interrogative or Oracle moves

The rules are exactly like in 4.2.5 anyway we would like to add the following remark: there is of course no reason for the Opponent to consult the oracle, he may asserts any atomic formulae he likes, and the Proponent will check the Opponent's assertion by consulting the oracle.

4.7 Structural rules

1. We will assume the intuitionistic structural rules
2. We will add the following rule :

(SR-Poker-Replay) (Replay): If a player X wins a first dialogical play for a sentence, another dialogical play for the same sentence should be played, where Y can change his choices but X has to follow the exactly same strategy (same choices at same or indistinguishable sequences of moves where the same information is available). The other player will play the new play being aware of this restriction of X. The player X is said to win the dialogue if and only if he wins both of these dialogical plays.

- It is important to notice that the notion *available information* involved in the SR-Poker-Replay rule implies that a player *never loses his own information.* Thus, in this context, given two quantifiers Q_1 and Q_2, it make no difference if they are or are not independent quantifiers provided that the player who has the choice is the same player for both. On the contrary the IF-Dialogues of 4.1 involve a different notion of availability because one player can *forget* some of his own previous moves. This makes victory in a replay harder to achieve. Indeed, in the case of dialogues of imperfect transmission of information, during a replay P must be able to find a strategy that yields victory independently even of the information of some of his own choices.

4.8 Let us play

Let us use the following notation for dialogues:

Like in 4.4 with the following difference:

- The second column of PUS contains expressions such as $x := c_1$ or $x := [i]$. The first signalizes that the Opponent has chosen the constant c_1 to instance the variable x. The expression $x := [i]$ signalizes a *closing the box* move.
- Analogous ca be said for the correspondent Proponent's column.

We will assume here the same assumptions and interpretations of the examples of 4.4.

Example 4.10 $\forall x\ \exists y\ \forall z/x, y\ \exists w/x, y\ R(x, y, z, w)$

The interpretation domain is $\{\delta_1, \ldots, \delta_i, \ldots, \delta_n\}$ *for any* $i \in \mathbb{N}^+$.

Let us take assume the following interpretation of the relation: $R : \{< \delta_i, \delta_j, \delta_k, \delta_l >$ *for any* $\delta_i, \delta_j, \delta_k, \delta_l$ *such that* $\delta_i, ...\delta_l \in Dom$ *and such that* $\delta_i = \delta_j\ \wedge\ \neg(\delta_k = \delta_l)\}$

Os				**Ps**			
$\xrightarrow{1}$	$[1] := c_1$		$\overrightarrow{15}$				
$\xrightarrow{5}$	$[5] := c_2$		$\overrightarrow{13}$				
O				**P**			
					$\forall x\ \exists y\ \forall z/x,y\ \exists w/x,y\ Rxyzw$		0
1	$x := [1]$	$?x := [1]$	0		$\exists y\ \forall z/x,y\ \exists w/x,y\ R[1]yzw$		2
3		$?\exists$	2		$\forall z/x,y\ \exists w/x,y\ R[1]f([1])zw$	$y := f([1])$	4
5	$z := [5]$	$?z := [5]$	4		$\exists w/x,y\ R[1]f([1])[5]w$		6
7		$?\exists$	6		$R[1]f([1])[2]g([5])$	$w := g([5])$	8
9		$?f([1])$	8		$R[1][1][5]g([5])$	$f([1]) := [1]$	10
11		$?g([5])$	10		$R[1][1][5][5]+1$	$g([5]) := [5]+1$	12
13		$open[5]$	12		$R[1][1]c_2c_3$		14
15		$open[1]$	14		$Rc_1c_1c_2c_3$		18
17		$Rc_1c_1c_2c_3$		15	$Oracle(Rc_1c_1c_2c_3, \neg Rc_1c_1c_2c_3)$		16

***P** wins.*

At this point the Opponent could start another play applying the Poker-Replay-Rule. The reader can easily verify that this will end up by a win of P.

Let us make explicit the moves of the play:

[$\xrightarrow{1}$ ***O** challenges the first quantifier choosing secretly the constant c_1 and hiding it in the box* [1].]

2 ***P** responds to the challenge by replacing x with* [1].

3 ***O** challenges the first existential quantifier.*

4 ***P** responds by introducing the function $f([1])$*

[$\xrightarrow{2}$ ***O** challenges the second universal quantifier choosing secretly the constant c_2 and hiding it in the box* [5].]

6 ***P** responds by replacing z with* [5].

7 ***O** challenges the last quantifier.*

8 ***P** responds by introducing the function $g([5])$*

9 ***O** asks for the definition of f.*

10 ***P** defines f as the identity function.*

11 ***O** asks for the definition of g.*

12 ***P** defines g as the immediate-successor function.*

[$\overrightarrow{13}$ ***O** demands the opening of the box* [5]

14 ***P** replaces the box* [5] *with its content and uses this information to give the concrete value of* $[5]+1$.

[$\overrightarrow{15}$ ***O** ask for the box to be open*]

15 ***O** demands the opening of the box* [1]

16 *As **P** can't assert an atomic formula before it has been conceded by O, P consults the oracle (makes an experiment, consults the model, uses an ultra-short-acting barbiturate as truth serum on **O**) to know which of the two literals holds: $Rc_1c_1c_2c_3$ or $\neg Rc_1c_1c_2c_3$.*

17 *The answer is that the positive literal holds and O concedes this explicitly*

18 ***P*** *uses this concession to defend the challenge made by* ***O*** *at move* 15*, and therefore, wins the dialogue.*

Actually, in order to test the truth of P's claim in the model a replay should be run using the Poker-Replay rule. We leave it to the reader.

Example 4.11 *Consider once more the truth of the formula of example 4.3:*

$$\forall x\ \exists y/x\ \neg(x = y) \tag{3}$$

Let us assume too the same domain as in example 4.3, namely: $\{\delta_1, \delta_2\}$.

Os				***Ps***			
$\overset{1}{\rightarrow}$	$[1] := c_1$		$\overset{\rightarrow}{6}$				
O				***P***			
					$\forall x\ \exists y/x\ \neg x = y$		0
1	$x := [1]$	$?x := [1]$	0		$\exists y/x\ \neg[1] = y$		2
3		$?\exists$	2		$\neg[1] = c_1$	$y := c_1$	4
5		$[1] = c_1$	4		$-$		
7		$c_1 = c_1$		5	$open[1]$		6
9		$c_1 = c_1$		7	$?oracle(c_1 = c_1, \neg c_1 = c_1)$		8

P *loses.*

Claim 4.12 ***P*** *loses in this dialogical play, but* ***O*** *does not have a winning strategy. The reader could easily verify this claim*

Example 4.13 (Signaling)

$$\forall x\ \exists z\ \exists y/x\ \neg(x = y) \tag{4}$$

Let us assume the following domain: $\{\delta_1, \ldots \delta_i, \ldots \delta_n\ \}$ *with* $\{1, \ldots i, \ldots n\} \subseteq \mathbb{N}^+$. *The following play is won by* **P***:*

Os				***Ps***			
$\overset{1}{\rightarrow}$	$[1] := c_3$		$\overset{\rightarrow}{12}$				
O				***P***			
					$\forall x\ \exists z\ \exists y/x\ \neg x = y$		0
1	$x := [1]$	$?x := [1]$	0		$\exists z\ \exists y/x\ \neg[1] = y$		2
3		$?\exists$	2		$\neg[1] = y$	$z := f([1])$	4
5		$?\exists$	4		$\neg[1] = g(f([1]))$	$y := g(f([1]))$	6
7		$[1] = g(f([1]))$	6		$-$		
9		$[1] = (f([1])$		7	$?g(f([1])) := f([1])$	$g(f([1])) := f([1])$	8
11		$[1] = [1] + 1$		9	$?f([1])) := [1] + 1$	$f([1]) := [1] + 1$	10
13		$c_3 = c_4$		11	$open[1]$		12
15		$\neg c_3 = c_4$			$oracle(c_3 = c_4, \neg c_3 = c_4)$		14
		-		15	$c_3 = c_4$		16

P *wins.*

Once more, the fact that the Proponent wins a dialogical play, does not mean that the thesis is true in the model, that is, that the Proponent has a "uniform winning strategy". To prove that such P has such a strategy the whole as to be replayed again as follows:

Os				***Ps***			
$\xrightarrow{1}$	$[1] := c_5$		$\overrightarrow{12}$				
O				***P***			
					$\forall x\ \exists z\ \exists y/x\ \neg x = y$		0
1	$x := [1]$	$?x := [1]$	0		$\exists z\ \exists y/x\ \neg[1] = y$		2
3		$?\exists$	2		$\neg[1] = y$	$z := f([1])$	4
5		$?\exists$	4		$\neg[1] = g(f([1]))$	$y := g(f([1]))$	6
7		$[1] = g(f([1]))$	6		$-$		
9		$[1] = (f([1])$		7	$?g(f([1])) := f([1])$	$g(f([1])) := f([1])$	8
11		$[1] = [1] + 1$		9	$?f([1])) := [1] + 1$	$f([1]) := [1] + 1$	10
13		$c_5 = c_6$		11	$open[1]$		12
15		$\neg c_5 = c_6$			$oracle(c_5 = c_6, \neg c_5 = c_6)$		14
		-		15	$c_5 = c_6$		16

P wins.

Example 4.14 (Dependent connective) *Assume the domain* $\{\delta_1, \delta_2, \delta_3\}$

Let us further assume the following interpretations: $P : \{<\delta_1, \delta_2>, <\delta_2, \delta_1>, <\delta_2, \delta_2>, <\delta_2, \delta_3>\}$ *and* $Q : \{<\delta_i, \delta_j>$: *for any* δ_i, δ_j *such that* $\delta_i = \delta_2$ *or* $\delta_i = \delta_3\}$.

Os				***Ps***			
$\xrightarrow{1}$	$[1] := c_2$		$\overrightarrow{11}$				
O				***P***			
					$\forall x\ (\exists y/x\ Pxy \vee \exists y/x\ \neg Pxy)$		0
1	$x := [1]$	$?x := [1]$	0		$\exists y/x\ P[1]y \vee \exists y/x\ \neg P[1]y$		2
3		$?\vee$	2		$f_\vee([1])$		4
5		$?f_\vee([1])$	4		$\exists y/x\ P[1]y$		8
7		$Q[1]c_1$		5	$?(Q[1]c_1, \neg Q[1]c_1)$		6
9		$?\exists$	8		$P[1]c_1$	$y := c_1$	10
11		$?open[1]$			Pc_2c_1		14
13		Pc_2c_1			$?oracle(Pc_2c_1, \neg Pc_2c_1)$		12

P *wins.*

Claim 4.15 ***P*** *wins in this dialogical game, but* ***P*** *does not have a uniform winning strategy.*

Proof.

P has a uniform winning strategy if and only if he can win in all dialogical plays. Let us undertake the brave task to show the whole play and replay:

	Os				***Ps***		
$\overset{1}{\to}$	$[1] := c_2$		$\overrightarrow{11}$				
$\overset{1'}{\to}$	$[1'] := c_3$		$\overrightarrow{11'}$				
	O				***P***		
					$\forall x\ (\exists y/x\ Pxy \vee \exists y/x\ \neg Pxy)$		0
1	$x := [1]$	$?x := [1]$	0		$\exists y/x\ P[1]y \vee \exists y/x\ \neg P[1]y$		2
3		$?\vee$	2		$f_\vee([1])$		4
5		$?f_\vee([1])$	4		$\exists y/x\ P[1]y$		8
7		$Q[1]c_1$		5	$?(Q[1]c_1, \neg Q[1]c_1)$		6
9		$?\exists$	8		$P[1]c_1$	$y := c_1$	10
11		$?open[1]$			Pc_2c_1		14
13		Pc_2c_1			$?oracle(Pc_2c_1, \neg Pc_2c_1)$		12
$1'$	$x := [1']$	$?x := [1']$	0		$\exists y/x\ P[1']y \vee \exists y/x\ \neg P[1']y$		$2'$
$3'$		$?\vee$	$2'$		$f_\vee([1'])$		$4'$
$5'$		$?f_\vee([1'])$	$4'$		$\exists y/x\ P[1']y$		$8'$
$7'$		$Q[1']c_1$		$5'$	$?(Q[1']c_1, \neg Q[1']c_1)$		$6'$
$9'$		$?\exists$		$8'$	$P[1']c_1$	$y := c_1$	10
$11'$		$?open[1]$					
$13'$		$\neg Pc_3c_1$		$11'$	$?oracle(Pc_3c_1, \neg Pc_3c_1)$		$12'$
$15'$		Qc_3c_1			$open$		$14'$
$17'$		Qc_3c_1		$11'$	$?oracle(Qc_3c_1, \neg Qc_3c_1)$		$16'$

***O** wins.*

■

The reader should be able to verify that actually the proponent has a winning strategy. Indeed he should ask $?(P[1]c_1, \neg P[1]c_1)$ rather than $?(Q[1]c_1, \neg Q[1]c_1)$.

Example 4.16 (Poker or IF) *Let us discuss once more the example 4.7*
$\forall y\ \exists z\ \exists v/y, z\ (Rc_1y\ \rightarrow\ (Ryz\ \wedge\ Rzv))$

provided by Tero Tulenheimo.

Let us recall our domain and interpretation function.
Domain:
$\{\delta_1, \delta_{2.1}, \delta_{2.2}, \delta_{3.1}, \delta_{3.2}, \delta_{3.3}, \delta_{3.4}, \delta_{4.1}, \delta_{4.2}, \delta_{43}\}$

Interpretation of R:
$\{< \delta_1, \delta_{2.1} >, < \delta_1, \delta_{2.2} >, < \delta_{2.1}, \delta_{3.1} >, < \delta_{2.1}, \delta_{3.2} >, < \delta_{2.2}, \delta_{3.3} >, < \delta_{2.2}, \delta_{3.4} >, < \delta_{3.1}, \delta_{4.1} >, < \delta_{3.2}, \delta_{4.2} >, < \delta_{3.3}, \delta_{4.2} >, < \delta_{3.4}, \delta_{4.3} >\}$.

The interpretation of c_1 is δ_1 and the interpretation of $c_{i.n}$ is $\delta_{i.n}$.

The reader can easily verify that there is a winning strategy for the Proponent - whatever structural rule we are playing with. Indeed, if the Opponent chooses $c_{2.1}$ let the Proponent choose $c_{3.2}$. If the Opponent chooses $c_{2.2}$ let the Proponent's choice be $c_{3.3}$ (if *the Opponent chooses one value for y which does not verify the antecedent the Proponent wins anyway). Then, let the Proponent's choice for v be* $c_{4.2}$.

This is in fact the same strategy as in example 4.7 but what we would like to point out here is that while playing with SR-Poker-Replay-rule the Proponent can also win with the following strategy where we assume that the Opponent chooses the values of y which verify the antecedent:
If the Opponent chooses $c_{2.1}$ *the Proponent chooses* $c_{3.1}$.
If the Opponent chooses $c_{2.2}$ *let the Proponent's choice be* $c_{3.4}$. *At this moment it is crucial to see that the Poker-Dialogues allow the Proponent to consult his own private information before choosing the appropriate value for v (namely:* $c_{4.1}$ *for* $c_{3.1}$ *and* $c_{4.3}$ *for* $c_{3.4}$*).*
Thus, the constant chosen by the Proponent to defend the first existential is not decisive, because he has later anyway access to his own private moves. This is certainly not the case with the dialogue described in 4.7, where the Proponent can forget the information of some of his previous choices. While playing with SR-Poker-Replay there is no difference between $\forall y\ \exists z\ \exists v/y, z\ (Rc_1y \rightarrow (Ryz \wedge Rzv))$ *and* $\forall y\ \exists z\ \exists v/y\ (Rc_1y \rightarrow (Ryz \wedge Rzv))$.

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