Dialogues and Defeasible Reasoning. Towards Modelling Disputes in Traditional African Law and Beyond

Shahid Rahman
(UMR: 8163, Universtié de Lille)

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1 ABSTRACT

In his challenging thesis of 2010 Gildas Nzokou showed that disputes in traditional Africal Law are from its logical structure not that different from those in Western cultures, if we take the standard modelisations of the latter developed by researchers such as Henry Prakken, however it adds a new perspective: a non monotonic reasoning perspective from the point of view of dialogues. The main result is that the study of African disputes offers new ways to understand defeasible reasoning within a dialogical structure. In fact, among the amazing results of Nzokou are:

(1) that the logical structure of debates in the context of African law correspon to some form of non-monotonic reasoning,
(2) that the notion of inference involved amounts to a dynamic and argumentative one. It is about logical consequence and not even about epistemic inference but about dynamic inference
(3) that the dynamic and argumentative features mentioned above can be modelled in the frame of a pragmatist semantics.

Actually the thesis of Nzokou contains some other challenging results such as on the structure of time and on the notion of *emptiness*.
I will not tackle these two issues however I will provide a general approach hat should provide a semantics rich enough to tackle these issues within the same framework. Moreover I will not take explicitly the examples of traditional law, in order to achieve generality. In fact the point that I would like to stress is that the structure of logical debates in African law can be used as general dynamic structure of non-monotonic reasoning: that is the legacy that I would like to highlight. In other words I would like to show how traditional and very well known cases and problems inherent in standard non-monotonic reasoning can be solved with what we learn from the logic of African debates.

2 Introduction

The term "non-monotonic reasoning" covers a family of formal frameworks devised to capture and represent *defeasible inference*, i.e., that kind of inference in which reasoners draw conclusions tentatively, reserving the right to retract them in the light of further information. Such inferences are called "non-monotonic" because the set of conclusions warranted on the basis of a given knowledge base, given as a set of premises, does not increase (in fact, it can shrink) with the size of the knowledge base itself. This is in contrast to standard logical frameworks (e.g., classical first-order) logic, whose inferences, being deductively valid, can never be "undone" by new information.

The most known of the examples is build around the poor *Tweety* that is a bird but can not fly.

> **Ex. 1**: Typically, it is assumed that birds fly. Thus, if a given animal (namely *Tweety*) is known to be a bird, and nothing else is known, it can be assumed to be able to fly. The default assumption must however be retracted if it is later learned that the considered bird is a penguin.

Nixon diamond:

> **Ex. 2**: Suppose our knowledge base contains (defeasible) information to the effect that a given individual, Nixon, is both a Quaker and a Republican. Quakers, by and large, are pacifists, whereas Republicans, by and large, are not. The question is what defeasible conclusions are warranted on the basis of this body of knowledge, and in particular whether we should infer that Nixon is a pacifist or that he is not pacifist. If there no reason to prefer either conclusion ("Nixon is a pacifist;" "Nixon is not a pacifist") to the other one, one kind of reasoner (usually called *credulous*, will definitely commit to one or the other. A different kind of reasoner, the s*ceptical* reasoner, recognizes that this is a conflict not between hard facts and defeasible inferences, but between two different defeasible inferences. Since the two possible inferences in some sense "cancel out," the sceptical reasoner will refrain from drawing either one.

A tricky example stems from Hans Rott

- **Ex. 3:** Imagine you are walking along a beach. It is a beautiful night and you are hungry. But you know that at the end of the beach there are two restaurants, one of the them run by Gildas, the other by Zorba. Now, you are still far away from the restaurants but you happen to perceive a shimmering light coming from that direction. This shimmering is enough to lead you to the belief that *either Gildas's or Zorba's restaurant is open* (Thus you assert that under such conditions you are willing to accept the conditional: *If Gildas's restaurant is not open, then Zorba's is*.($\sim p \rightarrow q$)). However, awhile approaching to the restaurants, you realize *that Gildas's restaurant is open and Zorba's is closed.* Consequently, it seems sensible to assume that you will withdraw the previous conditional (though perhaps you would not withdraw it if you understand the conditional not as indicative but as subjunctive: *If Gildas's restaurant had not open, then Zorba's would).* If we disallow the use of explosion the thesis will be no longer defendable in relation to the new information. Another way to analyse this failure is to involve subformulae of the thesis.

An important upshot of the latter way to see things is that *steps* leading to a thesis can be challenged and defeated by new information. Such kind of challenges build the core of the argumentative approach to defeasible reasoning such as in developed by John Pollock and Henry Prakken.

The analysis of defeasible reasoning within the dialogical framework should help to build a link between the approaches of default-logic and the argument-based approach initiated by John Pollock [1987], which defines notions like argument, counterargument, attack and defeat, and defines consequence notions in terms of the interaction of arguments for and against certain conclusions.

Gildas Nazokou developed in his thesis [2010] a dialogical approach to non-monotonic reasoning argumentation in African traditional law. The work of Nzokou provides important insights on the dialogical view on non-monotonic reasoning and motivated the further developments of the present paper. However, he uses belief-revision operators instead of defeasibility. Moreover, the difference between global and particle rules is not that neat (though it might perhaps be reformulated in that way).

1. Defeasibility and the semantics of dialogical logic

The standard logical approaches to model argumentation forms that involve defeasibility stress either the point on the non monotonic property of the turn-style or introduce a specific conditional (or a combination of the two). In a game-theoretic approach this means that either it is a property of winning strategies or it is about the rules that define a new logical constant (particle rules). In the present paper we will make use of an alternative to the approaches mentioned about available to the dialogical approach. The dialogical approach, as developed in the appendix, distinguishes between

- the rules defining the constants (*local meaning*),
- the rules that determine how to play (*global meaning*) and
- the rules that delineate the ways to win (*strategic level*) if winning is possible
- the notion of winning at the play level from the notion of winning at the strategic level.

In this context, defeasible reasoning is about introducing some restrictions on the rules on how to play (global meaning), it consists neither in introducing a new conditional (at the local level) nor in changing the winning-strategies. According to our view, defeasibility principle involves semantic features at the global level. More precisely, according to the dialogical approach, defeasibility amounts to the task of verifying that the proponent's thesis is compatible with the last concession(s) introduced in the play. Notice that the theory of meaning underlying dialogical logic provides a uniform semantics that though it is neither model-theoretic non-proof-theoretic can capture the features of both of those approaches within one frame.

Moreover, this approach allows to distinguish Pollock's *justification of a belief relative* to the actual defeasible reasoning at a given stage from *warrant* which assumes the set of all possible inferences that can be drawn by an idealized player at a given stage.[1] In the dialogical framework, while *warrant* corresponds to the strategic level (triggered by the extensive form of the play at a given stage) and justification corresponds to *the play level*. Precisely the dialogical level where the semantics is generated and where (as discussed in chapter 4 below) Pollock's notion of *multiple assignments* finds its natural place.

It should be also pointed out that the dialogical framework provides to the development of defeasible reasoning a level beyond the one of play and strategy: the level of *cycles* linked to the notion of *dialogue-defineteness*. Dialogical defineteness is the dialogical way to deal with Church's theorem on the non-decidability of first order logic. .

[1] Pollock [2008]. *Reasoning: Studies of Human Inference and its Foundations*, ed. Jonathan Adler and Lance Rips, Cambridge University Press. 451-470

1.1 Defeasibility and the Semantics of dialogical logic I: Particle rules

As already mentioned we will not introduce a particle rule for conditional, however we will make use of two operators, namely the *defendability* operator $\mathcal{V}$ and the *attackability* operator $\mathcal{F}$. These operators have been developed by Rahman and Rückert [2001] in a different context.[2]

The operator $\mathcal{V}$

	Challenge	**Defence**
X: $\mathcal{V}_{\mathcal{P}i}\,\varphi$	Y: ? $\mathcal{V}_{\mathcal{P}i}\,\varphi$	
	Subdialogue	*Subdialogue*
	(The challenger must play formally in the subdialogue)	X: $\varphi \wedge \mathcal{P}i$. (The defender chooses the subdialogue)

Notice that the upper dialogue and their subdialogues are sections of just one dialogical play where one of the argumentation partners wins or loses.

[2] Rahman introduced these operators in his Habilitationsschrift 1997 and were later on thorough studied by Rahman/Rückert ([2001]) in order to develop a dialogical semantics for connexive logic

In order to keep track of different parts of a dialogue play for defeasible argumentation we will distinguish:

> **Upper-section d_1 of a play:** that identifies the first -sections of a play where **P** defended a thesis given the initial concession(s) $\mathcal{P}i$.
>
> **Subdialogue $d_{i.n}$** of section **i**. The subdialogue has been triggered by the defending a $\mathcal{V}$ (or $\mathcal{F}$)-operator at section **i**.

The initial section (called also the initial dialogue) will be the section 1. It is assumed that, if the play continues after section 1, P wins at this section – otherwise the play stops there.

d_i Y	X
... ?$\mathcal{V}_{\mathcal{P}i}$	$\mathcal{V}_{\mathcal{P}i}\,\varphi$
$d_{i.m}$ Y	**X**
...	$\varphi \wedge \mathcal{P}i.$

Case 2:

d_i X	Y
... ?$\mathcal{V}_{\mathcal{P}i}$	$\mathcal{V}_{\mathcal{P}i}\,\varphi$
$d_{i.m}$ X	**Y**
...	$\varphi \wedge \mathcal{P}i.$

The operator $\mathcal{F}$

	Challenge	Defence
X: $\mathcal{F}_{\mathcal{P}i}\varphi$	Y: ? $\mathcal{F}_{\mathcal{P}i}$	
	Subdialogue	*Subdialogue*
	Y: $\mathcal{P}i\rightarrow\varphi$ φ (Y must defend φ). Y must play formally in the subdialogue	X: $\sim(\mathcal{P}i\rightarrow\varphi)$ X: $\mathcal{P}i$ (X challenges the conditional) The defender chooses the subdialogue

Again two cases (with and without changing the formal restriction) should be distinguished here:

Case 1:

d_i Y	X
?$\mathcal{F}_{\mathcal{P}i}$	$\mathcal{F}_{\mathcal{P}i}\ \varphi$
$d_{i.m}$ Y	**X**
. .$\mathcal{P}i\rightarrow\varphi$ φ	$\sim(\mathcal{P}i\rightarrow\varphi)$ $\mathcal{P}i$

Case 2:

d_i Y	X
… ?$\mathcal{F}_{\mathcal{P}i}$	$\mathcal{F}_{\mathcal{P}i}\ \varphi$
$d_{i.m}$ Y	**X**

$\mathcal{P}i \rightarrow \varphi$ φ	$\sim(\mathcal{P}i \rightarrow \varphi)$ $\mathcal{P}i$

1.2 Defeasibility and the Semantics of Dialogical logic II: Structural Rules

1.2.1 Sections of a play:

- Each section represents a part of the play where the Proponent defends the main thesis with **additional** concession(s) in relation to the concession(s) of initial section that starts the whole dialogue play.

- The whole play is unfolded in the following way

1 The dialogue starts with a main thesis and a concession, the play continues until the Proponent wins.

2 A second section of the dialogue starts with the same main thesis but with the addition of new concession(s), where the Proponent might lose.

3 In general, when a player looses on one section, he might start with a different one. In particular, if it is the opponent; he might start a new section (bigger than 1) by uttering:

O: $\mathcal{P}_i^{\alpha}$? (to be read: can you show that the thesis α is compatible with the additional concession $\mathcal{P}_i$?)

If it is the Proponent, he might start a new section by uttering:

P: $\mathcal{P}_j^{\alpha}$!) (to be read: I can show that the thesis α is (still) compatible with the additional concession $\mathcal{P}_j$)

- It the proponent wins the whole play, we say that the thesis in the context of the concessions of the first section is *undefeated*. Otherwise it *is defeated.*

- Sometimes we will call a conclusion with his supportive concession(s) an *inference.*

Structural rule for defeasibily III:

III.1 Acceptability of Concessions:

At any section different from the initial one, before defending the main thesis, the Proponent has the right to check the acceptability of the new concessions in relation to pre-established criteria. The criteria establish one order of importance in relation to the competing conclusions of new incoming concessions.

It is important to point out that these criteria might determined by an epistemic and/or socio-cultural background or they might be formal, such as **specificity** (e;g.: *Tweety is a penguin* is more specific than *Tweety is a bird*, and *Tweety is genetically modified penguin* is more specific than the other two)

The right procedure of checking must be determined in relation to these criteria.

In general we will assume the criteria are user-provided.

However we will develop our system in consideration of the criterion of specifity. Nzokou has argued convincingly that in traditional law this is the most frequent case

I will not make use here for the sake of simplicity of Nzokou"s important "Analogy operator". One can assume that before the specifity criteria is used the analogy operator has been implemented.

III.1a Acceptability of new concessions

Specificity:
If the criteria is specificity, and Y added a new concessions $\mathcal{P}_j$. X may ask Y to show that *at least one* of the new concessions entails *at least one* of the former ones $\mathcal{P}_i$ – assuming that they are different.
Accordingly, X might require Y to show that $\mathcal{P}_j. \rightarrow \mathcal{P}_i$ is the case (where "→" is the material implication). X expresses this requirement by uttering $\mathcal{F}(\mathcal{P}_j.\rightarrow \mathcal{P}_i)$. X must now defend formally the conditional that results from his own choice of which of the concessions of $\mathcal{P}_j$ will build the head of the conditional and which of the concessions of $\mathcal{P}_i$ will tail of the conditional.

- **Remark**: such checking subdialogues are not to overlap with the ones triggered by
  checking the compatibility of a new concession. In fact, the latter assume already that the new concession has been accepted

Take the old concessions to be $\mathcal{P}_i$ =: {*Tweety is a bird* (*Bt*), *All birds can Fly*($\forall$x(Bx→Fx)}, and the new concessions to be $\mathcal{P}_j$ =: {*Tweety is a penguin* (*Pt*), *All penguins are birds* ($\forall$x(Px→Bx)*, No penguin can fly* ($\forall$x(Px→~Fx)}. Thus, after X challenge$\mathcal{F}(\mathcal{P}_j.\rightarrow \mathcal{P}_i)$, Y will run a subdialogue showing that ((Pt∧$\forall$x(Px→Bx))→*Bt*) is indeed the case

III.1.b Acceptability of new concessions
Explosion:

Some complications might arise if the concessions are inconsistent. If the new concession is contradictory it will entail the second one by explosion.

If the initial concessions are contradictory then one can adopt three strategies, namely:

a) reject those concessions as fit to trigger defeasible reasoning,
b) accept the explosion and stop the argument there,
c) adopt some strategy to extract a consistent part (if there is any) of the concession at stake.

I think that in our context c is the most plausible case. A possible strategy for c is the following:

> The player X playing under the formal rule must utter all of his possible defences and all of those atomic formulae that his antagonist Y uttered to challenge formulae of the thesis. If X did not fulfil these conditions Y might require this fulfilment only after the the corresponding standard play finished.
>
> E.g.: If the pending defensive move involves the utterance of formula α. Y might state the following interrogative utterance: $?\alpha$

Accordingly explosion will not work since the conclusion will never be uttered.

III.1c *Unsolved conflicts*

Assume that we run two different dialogues in relation to one pre-agreed criterion C, such as specificity, for two conflicting inferences, one starting with the first inference of the Nixon diamond-example (mentioned in the introduction) and one starting with the second inference of this diamond. In both dialogues the proponent might reject each of the new concessions. In such a case , we say that the conflict is “unsolved”.

This adds an important feature to our analysis, namely the possibility to compare two different arguments.

We will discuss this point in chapter 4. Until then we will assume that all conflicts are not unsolvable in the sense specified above.

I think this case must be considered. It might well be that there are two premises with the same grade of specificity but that lead to incompatible conclusions. Probably in the real praxis one will be preferred over the other but a more general solution might be also designed.

2 Play level, Types of Ranks and Strategic Level

On of the main features of the dialogical approach lo logic is the distinction between play level and strategy level. Roughly the play level indicates on how to play, but not how to develop a winning strategy. The play level is the level in which you play against a real player with all his skills and limitations. This level seems to be crucial in the study of defeasible reasoning. As already mentioned in the introduction to chapter 1 this seems to correspond to Pollock's [2008] notion of *justification* of an inference relative to a given stage. Certainly, this does not exclude a strategic level – Pollock's notion of *ideal warrant* - that defines an appropriate notion of defeasible validity, as we will discuss below.

Accordingly we will introduce a typical dialogical device: ranks. We will make use of two kind of ranks, namely, **S** ranks (section-ranks) and **R**-ranks (repetition ranks). The **S**-rank fixes the number of sections that may define a a whole play. Now, in general we assume that this rank is bigger than three and is imposed from outside. In this context this seems to be reasonable. External limits such as time or deadlines usually determine the length of the concatenation of arguments and counterarguments - – external ranks for non-monotonic reasoning have been used for the first time by Nzokou [2010].

An important related issue is the free choice of the concessions: We can assume that the choice of concessions is a choice of the players during the play (let us call them *internal choices of concessions*) but limited by the choice within some background finite source (such as in law) or internal and infinite or that there is a dynamic external procedure that provides the concessions (*external choices of concessions*). The background source of concessions could be thought as a kind of oracle that either provides at different stages of the play different concessions or allows to choose some of them. [3] This source is not to be thought as a set of concessions from which the inferences will be drawn but rather as a dynamic system that provides the set of concessions form which inferences will be drawn *if these concessions are acceptable*.

One reasonable idea is to combine external choices of **S**-ranks with a limited internal source. A strategy that I think captures a good deal of defeasible argumentation forms. Indeed, assume that the **S**-rank is *m*, and within the range of this scope, and in relation to the (acceptable) concessions provided by the source the proponent always wins. Then; we can distinguish at *m* the play level, that is the level at which the proponent won his thesis in relation to the actual moves of the opponent; and the strategic level. The level at which each of the possible relevant moves of the opponent have been considered at *m*. Pollock [2008], calls this the *diachronically fixed ideal warrant* (here fixed at level *m*).

[3] This is linked to the Hintikka's uses of the oracle in the context of enquiry games (cf. Hintikka/Halonen/Mutanen 1988, 47-90).

Now, as rightly stressed by Pollock [2008] because of Church's theorem on the undecidability of first-order logic, the ideal warrant might never be reached. Here we will briefly introduce an additional notion of rank that allows to distinguish a further level. In dialogical logic we speak of repetition ranks (**R**-ranks) in the following way:

- After the move that sets the thesis players **O** and **P** each choose a natural number n_i and n_j espectively (termed their repetition ranks). Thereafter the players move alternately, each move being a request or an answer.

- In the course of the dialogue, **O** (**P**) may attack or defend any single (token of an) utterance at most n_i (or n_j) times.

- This notion of **R**- rank assumes that the attacks –and defences, are of the same kind. Recall that in our context there might be three kinds of challenge: (i) those determined by the involved particles, (ii) the compatibility-challenges $\mathcal{P}_i^{\alpha}?$, $\mathcal{P}_i^{\alpha}!$, (iii) the acceptability challenge. Thus, if the **R**-rank is 1 one formula might be challenged once with each of the kinds of challenge just mentioned. ,

The **R**-rank, determine the level of *cycles* that can be played at given **S**-rank. Take the **S**-rank to be two, if the concessions involves quantification the number of plays at **S**-rank 2 might be infinite. However at this level we could further distinguish the number or cycles to be played at the fixed S-rank 2: at is the number n_i (or n_j) of times that a given formula can be challenged or defended at play the S-rank of which is 2.

Example 1:

Let us display here a very well known example:

We assume that the **S**- and the **R**-rank is 2:

𝒫1: Pigs do not fly (∀x (Px→~Fx)). Babe is a pig. (Pb)
Thesis: Babe does not fly

𝒫2:: Genetically modified pigs can fly (∀x(Px∧Gx→Fx)). Babe is a genetically modified pig (Pb∧Gb).

d_1 **O**		**P**	
𝒫1.1: ∀x(Px→~Fx)		~Fb	0
𝒫1.2: Pb			
1 Fb	0	*𝒫1.1* ?*b*	2
3 Pb→~Fb		3 Pb	4
5 ~Fb		5 Fb	6
—			
7 ? $\mathcal{P}2^{\sim Fb}$	0	𝒱(~Fb∧ 𝒫2)	8
9 ? 𝒱	8		

$d_{1.1}$ **O**		**P**	
		~Fb∧ *𝒫2*	10
11?∧2	10	*𝒫.2.1:* ∀x(Px∧Gx→Fx)	12
13?∧1	*𝒫.2.2:* 12	*𝒫.2.2:* Pb∧Gb	12
15?∧2	*𝒫.2.2:* 12	Pb	14
17?*b*	*𝒫.2.1:* 12	Gb	16
19?-Pb∧Gb	20	Pb∧Gb→Fb	18
21?∧1	12	Fb	20
23?-Fb	22	~Fb	22
		—	
		The proponent looses and thus the inference has been defeated	

Defeasibility and the Challenge of Intermediate Steps:

Let us delve into Roth's example mentioned in the introduction , that requires a more general approach to compatibility.

P1.: *Either Gildas's or Zorba's restaurant is open* ($p \vee q$)

P2. Gildas's restaurant is open (p), *Zorbas restaurant is not open* (~q)

Thesis: *If Gildas's restaurant is not open, then Zorba's is* . (~p→q)

Interesting is that, if we allow explosion, the thesis is compatible with *P2.* Let us assume that we do not allow explosion using the acceptability device c) mentioned at III.1b, namely

> The player X playing under the formal rule must utter all of his possible defences and all of those atomic formulae that his antagonist Y uttered to challenge formulae of the thesis. If X did not fulfil these conditions Y might require this fulfilment only after the the corresponding standard play finished.

In this case, the proponent will loose since, when required to utter *q* (see move 17 below) he will loose. Notice that even with this device, the proponent wins the thesis in the context of *P1.* Indeed, the proponent, can win since the opponent will utter both of the disjuncts in the same play. Certainly the use of this device takes us closer to relevant logic.

d_1	**O**			**P**	
$\mathcal{P}1$:	$p \vee q$			$\sim p \rightarrow q$	0
1	~p	0		q	6
3	p		$\mathcal{P}1$	$? \vee$	2
	—		1	p	4
5	q				
7	? $\mathcal{P}2^{\sim p \rightarrow q}$	0		$\mathcal{V}\sim p \rightarrow q \wedge \mathcal{P}2$	8
9	? $\mathcal{V}$	8			

$d_{1.1}$	**O**			**P**	
				$\sim p \rightarrow q \wedge \mathcal{P}2$	10
11	$? \wedge 2$	10		$\mathcal{P}.2.1$: p	12
				$\mathcal{P}.2.2$: $\sim q$	12
13	$? \wedge 1$	10		$\sim p \rightarrow q$	
15	$\sim p$	14	14		
	—			q	18
17	$?\text{-}q$			p	16
14					
19	q	$\mathcal{P}.2.2$: 12		—	
				The proponent looses and thus the inference has been defeated	

Another way to analyse this failure is to involve subformulae of the thesis. Indeed, while defending the thesis *~p→q* (given the premise $p \vee q$) the following is possible: The thesis is grounded by either *p* and/or *q*. If the first then the thesis can be inferred by explosion. If the second is the case, then the tail of the condition- that constitutes the thesis- will be the case and thus ground the thesis. However *q* is obviously incompatible with the new premise *~q*.

- An important upshot of this way to see things is that *steps* leading to a thesis can be
  challenged and defeated by new information. Such kind of challenges build the core of the argumentative approach to defeasible reasoning of John Pollock and Prakken above.

This might motivate the following extension

1 The dialogue starts with a main thesis and a concession, the play continues until the Proponent wins.

2 A second section of the dialogue starts with the same main thesis but with the addition of new concession(s), where the Proponent might lose.

3 In general, when a player looses on one section, he might start with a different one. In particular, if it is the opponent; he might start a new section (bigger than 1) by uttering:

> **O:** $\mathcal{P}_i^{\alpha}$? (to be read: can you show that the thesis α is compatible with the additional concession $\mathcal{P}_i$?) *and/or*
>
> **O:** $\mathcal{P}_i^{\beta!}$? (to be read: can you show that the defensive move β involved in the defence of α is compatible with the additional concession $\mathcal{P}_i$?)

If it is the Proponent, he might start a new section by uttering:

P: $\mathcal{P}_j^{\alpha}$!) (to be read: I can show that the thesis α is (still) compatible with the additional concession $\mathcal{P}_j$) *and/or*

P: $\mathcal{P}_i^{\beta!}$! (to be read: I can show that the defensive move β is compatible with the additional concession $\mathcal{P}_i$). This second possibility assumes that the opponent challenged the compatibility of the defensive move in an upper section of the play.

This, form might shorten some plays at the level of the subdialogue. Indeed let us see how it works in relation to Rott's example:

d_1	**O**			**P**	
$\mathcal{P}1$:	$p\vee q$			$\sim p\rightarrow q$	0
				q	6
1	~p	0	$\mathcal{P}1$	?∨	2
3	p —		1	p	4
5	q			$\mathcal{V}q\wedge\mathcal{P}2$)	8
7	? $\mathcal{P}2^{q}$	0			
9	? $\mathcal{V}$	8			

$d_{1.1}$	**O**			**P**	
				$q\wedge\mathcal{P}2$	
11	?∧2	10	10	$\mathcal{P}.2.1$: p	
13	?∧1	10	12	$\mathcal{P}.2.2$: ~ q	
15	q	12	12	q —	14
				The proponent looses	

Nixon Diamond, Defeasibility and Super-Criteria in the Dialogical Framework

Let us briefly recall once more Nixon's diamond problem.

> Suppose our knowledge base contains (defeasible) information to the effect that a given individual, Nixon, is both a Quaker (Qn) and a Republican (Rn). Quakers, by and large, are pacifists ($\forall x(Qx \rightarrow Px)$), whereas Republicans, by and large, are not ($\forall x(Rx \rightarrow \sim Px)$).
>
> These allows the following two inferences with contradictory concessions:
>
> 1) Infer Pn from $\forall x(Qx \rightarrow Px)$ and Qn
> 2) Infer ~Pn from $\forall x(Rx \rightarrow \sim Px)$ and Rn

As mentioned above, if there id no reason to prefer either conclusion ("Nixon is a pacifist;" "Nixon is not a pacifist") to the other one, one kind of reasoner (usually called *credulous*, will definitely commit to one or the other at random (or both alternatively) and further explore its consequences. A different kind of reasoner, the s*ceptical* reasoner, will refrain from drawing either one. The sceptical approach is often defended by saying that since in an unresolvable conflict no argument is stronger than the other, neither of them can be accepted as justified, while the credulous approach has sometimes been defended by saying that the practical circumstances often require a person to act, whether or not this person has conclusive reasons to decide which act to perform.[4]

[4] Prakken and Vreeswijk give a more fine-grained picture of these antagonist positions. Cf. Prakken and Vreeswijk [2001], chapter 4.3.

Notice neither of the concessions is more specific than the other. Thus, if we start with one the second could be rejected. But this works too if we start the other way cycle. If there were a third concession that defeats one of the conclusions and this concession satisfy some agreed criterion; then a solution is possible since the non defeated will be re-established. For example take that the criterion is specifity and we have a third concession that states that “Quakers who live in Chicago are not pacifists” and “Nixon leaves in Chicago” then it is apparent that the inference 2) has been re-established.

But what happens if this is not the case. Well, we can stick to our criterion and hold that since there is no reason to prefer one of them over the other we refrain from both.

Reiter [1980] and Dung [1995] have proposed a solution with help of the notion of possible extensions:

- Given a set Γ of defaults, an extension for Γ represents a set of inferences that can be *reasonably* and *consistently* drawn using defaults from Γ

The idea is, roughly, then to endorse a conclusion φ if and only if φ is contained in *every* extension of the theory. Since this will not be possible in the case of the Nixon diamond nothing can be concluded.

Pollock [2008] produced some counterexamples and Makinson [1994] observed that sceptical consequences based on the notion of extension fail to be cautiously monotonic.

One first approach would be to follow assign arbitrary one criterion Ci that yields that one of both inferences, say inference 1, is defeated and proceed further on. If the further development shows that according to this criterion Ci the inference 1 is defeated after all, then start again with the second inference. The point is that this approach considers each inference individually and not a whole that interacts. The latter motivates Pollock's notion of *multiple assignements.* Pollock's idea is to introduce the idea of partial multiple assignments – that have some echoes of van Fraassen's supervaluations, where an assignment is a partial function defined over the set {defeated, undefeated} to steps of an argument: Instead of studying all the possible inferences, the idea is to sudy the possible assignments to some steps in the inference. The interesting point is that this super-assignment technique can be performed at the play level. Accordingly,

> Given a set of arguments X and a relation of defeat on X an argument is *justified* iff it receives the assignement 'defeated' in all status assignments to X.

We will provide only the first steps towards implementing these ideas by desgining two kinds of dialogues, where the first is closer to the credulous and the second closer to the sceptical approach:

Assume that we run two different dialogues in relation to one pre-agreed criterion C, such as specificity, for two conflicting inferences, one starting with the first inference of the Nixon diamond and one starting with the second inference of this diamond. In both dialogues the proponent might reject each of the new concessions, thus, as pointed out in III.1c (chapter 1.2.4) the conflict is *unsolved.*

Super-Criteria:

1)If, according to one pre-agreed criterion the conflict between two inferences is unsolved, then let the proponent chose a criterion for the acceptability of a new concession in order to test the thesis or to some defensive steps of the thesis. If, according to the criterion, the proponent wins then the first inference is said to be undefeated. Otherwise it has been defeated.

2)If, according to one pre-agreed criterion the conflict between two inferences is unsolved, then let the opponent chose a criterion for the acceptability of a new concession in order to test the thesis or to some defensive steps of the thesis. If, according to the criterion, the opponent wins the we say that conflict has been defeated. Otherwise it is undefeated.

We assume here the approach that allows to challenge defensive steps developed in the preceding chapter.

Conclusions and the way ahead:

From the dialogical point of view, defeaseable reasoning is about a global rule that establishes the compatibility of a thesis or steps defending the thesis in the context of some concessions. It's semantics is situated at the play-cycle level and this provides its dynamic features. Moreover, according to our view the approach allows to combine the "logical" with the "argumentative" approaches

There is quite a big deal work to do, let us enumerate some pressing ones:

1) To develop further the approach in order to incorporate the interaction of different arguments but also between arguments and subarguments
2) To compare the advantages and disadvantages of the dialogical approach to multiple assignments with the one of Pollock
3) To study the case of self-defeating arguments: that is those arguments where a step of an argument conflict with the conclusion of the same argument. In particular if an how to exempt self-defeating steps from multiple-assignments.
4) To re-consider the approach in the context of a limited choice of possible inferences.
5) To study the fruitfulness of the present approach in view Nzokou's challenging proposals mentioned above.

Appendix: Dialogical Logic

1. Introduction:

Dialogical logic (DL) searches both to recover the philosophical and technical link between argumentation and logic (logic as Agon) via the development of a *pragmatist semantics*. This semantics provides the basis for the notion *of formal strategy* by the means of which inference is understood dynamically, i.e., as a kind of a rational interaction of agents.

Developed by Paul Lorenzen and Kuno Lorenz DL was the result of a solution to some of the problems that arouse in Lorenzen' *Operative Logik* (1955).[5] As pointed out by Peter Schroeder-Heister, the insights of Operative logic had lasting consequences in the literature on proof-theory and still deserve attention nowadays.[6] Moreover, the notion of *harmony* formulated by the antirealists and particularly by Dag Prawitz has been influenced by Lorenzen's notions of *admissibility*; *eliminability* and *inversion*. However, on my view, the dialogical tradition is rather a rupture than a continuation of the operative project and it might be confusing to start by linking conceptually both projects together.

Historically speaking, dialogical logic, was suggested at the end of the 1950s by Paul Lorenzen and then worked out by Kuno Lorenz.[7] Inspired by Wittgenstein's *meaning as use* the basic idea of the dialogical approach to logic is that the meaning of the logical constants is given by the norms or rules for their use. This feature of its underlying semantics quite often motivated the dialogical approach to be understood as a *pragmatist* semantics.[8]

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[5] Cf. Lorenz 2001.
[6] Schröder-Heister 2008.
[7] The main original papers are collected in Lorenzen/Lorenz 1978. A detailed account of recent developments can be found in Felscher 1985, Keiff 2004, Keiff 2007, Rahman 2009, Rahman/Keiff 2004, Rahman/Clerbout/Keiff 2009, Keiff 2009, Fiutek/Rückert/Rahman 2010, Rahman/Tulenheimo 2006, Rückert 2001, Rückert 2007. For a textbook presentation (in French), see Fontaine/Redmond 2008.
[8] Quite often it has said that dialogical logics has a *pragmatic* approach to meaning. I concede that the terminology might be misleading and induce one to think that the theory of meaning involved in dialogic is not semantics at all. Helge Rückert proposes the more appropriate formulation *pragmatistische Semantik* (*pragmatist semantics*).

The point is that those rules that fix meaning may be of more than one type, and that they determine the kind of reconstruction of an argumentative and/or linguistic practice that a certain kind of language games called dialogues provide. As mentioned above the dialogical approach to logic is not a logic but a semantic rule-based framework where different logics could developed, combined or compared using the same logical and semantic "terminology".

In a dialogue two parties argue about a thesis respecting certain fixed rules. The player that states the thesis is called Proponent (**P**), his rival, who puts into question the thesis is called Opponent (**O**). In its original form, dialogues were designed in such a way that each of the plays end after a finite number of moves with one player winning, while the other loses. Actions or moves in a dialogue are often understood as *utterances*[9] or as speech-acts[10]. The point is that the rules of the dialogue do not operate on expressions or sentences isolated from the act of uttering them.[11]. The rules are divided into particle rules or rules for logical constants (*Partikelregeln*) and structural rules (*Rahmenregeln*). The structural rules determine the general course of a dialogue game, whereas the particle rules regulate those moves (or utterances) that are requests (to the moves of a rival) and those moves that are answers (to the requests).

Crucial for the dialogical approach, and that distinguishes it from all other approaches are the following points (that will motivate some discussion further on)

1. The distinction between local (rules for logical constants) and global meaning (included in the structural rules) – it is not the difference between introduction-elimination rules and structural rules.
2. The player independence of local meaning (recall that in a tableaux system, for example, the meaning of the logical constants is dependent on the sides: True-rules and False-Rules)

[9] Cf. Rahman/Rückert 2001, 111 and Rückert 2001, chapter 1.2.
[10] Cf. Keiff 2007.
[11] Tulenheimo 2010.

3. The distinction between the play level (local winning or winning of a play) and the strategic level (global winning; or existence of a winning strategy) – the notion of play does not correspond neither to winning in a model nor to a branch in branch proof_tree)
4. the notion of formal play and strategy (this does not correspond to true in any model, but true in a play where the proponent does not know about the justification of the atomic formulae)

2. Dialogical Logic and Meaning

Our aim here is to build the conceptual kernel of dialogic in the context of the dialogical reconstruction of first order propositional calculus, in its classical and intuitionist versions. There are many different formulations we will follow here essentially the notations and definitions given by Nicolas Clerbout in a forthcoming paper on the metalogic of Dialogical logic

Let our language $\mathcal{L}$ be composed of the standard components of first order logic with

connectives $\wedge$, $\vee$, $\rightarrow$, $\neg$, and two quantifiers $\forall$, $\exists$ – the conjunction might be indexed yielding $\wedge_i$, where $i \in \{1, 2\}$, such that "$\wedge_1$" stands for the first conjunct from left to right,
small letters (*p*, *q*, …) for atomic formulæ,
greek letters (α, β, …) for formulæ that might be complex,
capital italic bold letters (***A***, ***B***, ***C***, …) for predicators,
constants k_i, where $i \in N$, and
variables *x*, *y*, *z*,...

We will also need the following special symbols and idioms:

Force symbols: ? and !. The interrogation sign might be combined by means of specific rules yielding: $?\wedge_i$, $?\vee$, $?_{ki}$, $?_\exists$
The sign "?" ("!") signalises that a given move is a challenge (defence).

Rank-idioms: $\boldsymbol{r}$:= N, $\boldsymbol{r'}$: = N' where N and N' are natural numbers and "$\boldsymbol{r}$" ("$\boldsymbol{r'}$") stand for "repetition rank". Accordingly, "$\boldsymbol{r}$ = 1" and "$\boldsymbol{r'}$ =2" signalise that the repetition ranks chosen are 1 and 2.

Def. 1: An *expression* of $\mathcal{L}$ is either a term, a formula, a force symbol or a rank idiom.

Def. 2: Every expression *e* of our language can be augmented with labels **P** or **O** (written **P**-*e* or **O**-*e*, called (*dialogically*) *signed expressions*), meaning in a game that the expression has been uttered by **P** or **O** (respectively). We use X and Y as variables for **P**, **O**, always assuming X≠Y.

Def. 3: A move μ is dialogically signed expression X-*e*.

2.1 Local Meaning

Particle rules:

In dialogical logic, the particle rules are said to state the *local semantics*: what is at stake is only the request and the answer corresponding to the utterance of a given logical constant, rather than the whole context where the logical constant is embedded.

- The standard terminology makes use of the terms *challenge* or *attack* and *defence*.[12]

However let me point out that at the local level (the level of the particle rules) this

terminology should be devoid of strategic underpinning.

- *Declarative utterances* involve the use of formulae, *interrogative utterances* do not involve the use of formulae

[12] See Keiff 2007, Rahman/Clerbout/Keiff 2009. Tero Tulenheimo pointed out that this might lead the reader to think that already at the local level there are strategic features and that this contravenes a crucial feature of the dialogical framework. Indeed, Laurent Keiff [2007] introduced the terminology *requests* and *answers*. However the dialogical vocabulary has been established with the former choice and it would be perhaps confusing to change it once more.

The following table displays the particle rules, where X and Y stand for any of the players **O** or **P**:

$\vee$, $\wedge$, $\rightarrow$, $\neg$, $\forall$, $\exists$	**Challenge**	**Defence**
X: $\alpha\vee\beta$	**Y**: ?-$\vee$	**X**: α *or* **X**: β (**X** chooses)
X: $\alpha\wedge\beta$	**Y**: ?$\wedge$1 or **Y**: ?$\wedge$2 (**Y** chooses i $\in$ {1, 2})	**X**: α *respectively* **X**: β
X: $\alpha\rightarrow\beta$	**Y**: α (**Y** challenges by uttering α and requesting *B*)	**X**: *B*
X: $\neg\alpha$	**Y**: α	— (no defence available)
X: $\forall x\alpha$	**Y**: ?*k* (**Y** chooses)	**X**: α [*x*/*k*]
X: $\exists x\alpha$	**Y** ?$\exists$	**X**: α [*x*/*k*] (**X** chooses)

In the diagram, α[*x*/*k*] stands for the result of substituting the constant *k* for every occurrence of the variable *x* in the formula α.

One interesting way to look at the local meaning is as rendering an abstract view (on the semantics of the logical constant) that distinguishes between the following types of actions:

a) Choice of declarative utterances (=:disjunction and conjunction).
b) Choice of interrogative utterances involving individual constants (=: quantifiers).
c) Switch of the roles of defender and challenger (=: conditional and negation). As we will discuss later on we might draw a distinction between the switches involved in the local meaning of negation and the conditional).

Let us briefly mention two crucial issues

- **Player independence:** The particle rules are symmetric in the sense that they are player independent – that is why they are formulated with the help of variables for players. Compare with the rules of tableaux or sequent calculus that are asymmetric: one set of rules for the *true(*left)-side other set of rules for the *false(*right)-side. The symmetry of the particle rules provides, the means to get rid of tonk-like-operators.
- **Sub-formula property**: If the local meaning of a particle # occurring in φ involves declarative utterances, these utterances must be constituted by sub-formulae of φ.[13]

2.2 Plays and Games

Def. 4:
A *play* is a legal sequence of moves that complies with the moves of the particle rules described above and the structural rules to be described below.

Def. 5:
The *dialogical* game for α ($\mathcal{D}$α) is the set of all plays with thesis α in the sense of the starting rule SR-0 (given below).

Def. 6:

[13] This has been pointed out by Laurent Keiff and by Helge Rückert in several communications.

A *X-terminal play for α* is a play Δ in $\mathcal{D}\alpha$ such that the last member of Δ is a X-move and there is no Y-move μ such that $\Delta' = \Delta \frown \mu$ is a play in $\mathcal{D}\alpha$ - where "$\Delta \frown \mu$" stands for a play that extends Δ with the move μ.

A *dialogue for α* is a X-(or Y)-terminal play

Def. 7:
For any sequence Σ of moves μ, the function π^{Σ} assigns to each member of Σ a (non-negative) *position (-number)*: if μ is the *i*-th member of Σ, then $\pi^{\Sigma}(\mu) = i-1$. Thus, if μ is the first member of the sequence the function will assign this move the position 0. (if there is no ambiguity on which is the sequence involved we will write simply π^{Σ}.)

We will also make use of a further function defined by Felscher (1985):

Def. 8:
For any sequence Σ of moves μ, the partial function $\mathcal{J}^{\Sigma}$ assigns to each member of Σ, that is not a rank idiom (***r***=:N, ***r'***=N′) and that has a *position* bigger than 0 a pair [μ', Z] such that $Z \in \{?, !\}$, μ' is a move of the antagonist player and $\pi^{\Sigma}(\mu') < \pi^{\Sigma}(\mu)$.
The intended interpretation of $\mathcal{J}^{\Sigma}(\mu)$ is that each move μ of the sequence that is neither the thesis (that has position 0) nor is a rank idiom either challenges a previous move μ' or is a defence against a previous challenge μ', where μ' is a move of the antagonist player.

2.3 Global Meaning

Structural rules:

(SR-0) (starting rule):

Every play in the *dialogical game* for α ($\mathcal{D}\alpha$) starts by a move of **P** uttering α such that its position is 0. It provides the topic of the argumentation and is called the *thesis* of the play.

Moves are alternately uttered by **P** and **O**. That is, given a play Δ in $\mathcal{D}\alpha$ and a move μ in Δ, it is the case that if π(μ) is even then it is a **P**-move. Dually moves with odd positions are **O**-moves.

Comment: The proviso *if possible* relates to the utterance of atomic formulae. See formal rule *(SR* 2*)* below.

(SR-1) (no delaying tactics rule):

Both **P** and **O** may only make moves that change the situation.

After the move that sets the thesis players **O** and **P** each choose a natural number ***r*** and ***r'*** respectively (termed their *repetition ranks*).

In the course of the dialogue, **O** (**P**) may attack or defend any single (token of an) utterance at most ***r*** (or ***r'***) times.

Notice that the repetition rank does apply neither to the move that fixes a repetition rank nor to the utterance of the thesis: it fixes only the number of utterances of a move that is a challenge or a defence.

Thus, each move whose position is bigger than 2 is either a challenge or a defence (see Def-8) – since at position 0 the thesis is uttered and positions 1 and 2 are occupied by moves that result from the choice of a repetition rank.

(SR-2) (formal rule):[14]

P may not utter an atomic formula unless **O** uttered it first.

[14] See a discussion of this rule below in the commentaries about the dialogical notion of validity

More precisely, a sequence Δ in the dialogical game $\mathcal{D}\alpha$,where the thesis α is a complex formula, constitutes a *formal play* for α if for any atomic formula *q* in Δ it is the case that

If $\mu = \mathbf{P}\text{-}q \in \Delta$, then there is a $\mu' = \mathbf{O}\text{-}q \in \Delta$ such that $\pi^{\Delta}(\mu') < \pi^{\Delta}(\mu)$.

Atomic formulae can not be challenged (i.e., for any atomic formula *q* occurring in a play Δ there is no move in that play of the form [*q*, ?])

The dialogical framework is flexible enough to define the so-called *material dialogues*, that assume that atomic formulae have a fixed truth-value:

*(SR *2) (rule for material dialogues):*
Only atomic formulae standing for true propositions may be uttered. Atomic formulae
standing for false propositions can not be uttered.

(SR 3) (winning rule):
X wins iff it is Y's turn but he cannot move (either challenge or defend).

More precisely, X wins a play Δ for α in $\mathcal{D}\alpha$ iff it is X-terminal (see Def-6)

Global meaning
These rules determine the meaning of a formula where a particle occurs as a main operator in every possible play.

(SR 4c) (classical rule):

In any move, player X (Y) may challenge a complex formula uttered by his partner or he may defend himself against any challenge (including those challenges that have already been defended once) at most ***r***-times (***r'***-times).
or
(SR 4i) (intuitionist rule):[15]
In any move, player X (Y) may challenge a (complex) formula uttered by his partner at most ***r***-times (***r'***-times) - where ***r*** (***r'***) is the correspondent repetition rank - or he may defend himself *against the last challenge that has not yet been defended* – the latter condition on defences has priority over ***r.*** (see example 1).

- In fact; for both rules we will assume that **O** always chooses repetition rank 1.
- The intuitionistic rule might prevent **P** to apply a repetition rank ***r*** >1 to a defence.

Indeed, if P chooses ***r*** := 2 and there is a **P**-move that has already been defended,
according to SRè4i he will not be able to defend it once more. (see example 1).

- Notice that the dialogical framework offers a fine-grained answer to the question: Are
intuitionist and classical negation the same negations? Namely: The particle rules are
the same but it is the global meaning that changes.

In the dialogical approach validity is defined via the notion of *winning strategy*. Informally, a winning strategy for X means that for any choice of moves by Y, X has at least one possible move at his disposal such that he (X) wins:

Validity (definition):
A formula is valid in a certain dialogical system iff **P** has a formal winning strategy for this formula.

[15] In the standard literature on dialogues, there is an asymmetric version of the intuitionist rule, called E-rule since Felscher [1985].

Thus,

- α is classically valid if there is a winning strategy for **P** in the dialogical game $\mathcal{D}_c\alpha$
- α is intuitionistically valid if there is a winning strategy for **P** in the dialogical game $\mathcal{D}_i\alpha$.

Before we tackle the notion of strategy more thoroughly let us point out some features of the dialogical notion of validity and display then some examples:

Comments on the dialogical notion of validity:

Helge Rückert (2011) pointed out, and rightly so, that the formal rule triggers a novel notion of validity.[16]. Validity, is not being understood as being true in every model, but as *having a winning strategy independently of any model* or more generally independently of any *material* grounding claim (such as truth or justification). The copy-cat strategy implicit in the formal rule is not copy cat of groundings but copy-cat of declarative utterances involving atomic formulae. Moreover one should add that there is the notion of *formal play*, that does not seem to correspond to nothing in model-theoretic approach: a formal play is not playing in a model.

In fact, one could see the formal rule as process the first stage of which starts with what Laurent Keiff called *contentious* dialogues.[17] Contentious dialogues are dialogues where a player X utters one or more atomic formulae that are dependent upon a given ground and X is not prepared to put this ground into question – one can think of it as a claim of having some kind of ground (or a claim of truth) for it. Moreover the antagonist is willing to concede this ground for the sake of the argument.[18] Now, if we would like to avoid to have the result that an atomic formula is true by the only

[16] Talk at the worshop Proofs and Dialogues, Tübingen, Wilehm-Schickard Institut für Informatik, 25-27; February 2011/.
[17] Cf. Clerbout/Keiff/Rahman 209 and in Keiff/Rahman 2010.
[18] Cf. Keiff/Rahman 2010 (156-157), where this is linked to some specific passages of Plato's *Gorgias* (472b-c).

reason that the player X stated it – that is, if we want to find a way out of contentious dialogues, then there are two possible ways:

- either we accept some principle of grounding external to the dialogue itself (and thus external to the interaction of the players)

  or
- we look for a player principle of grounding that is internal to the dialogue and dependent on the interaction of the players.

The first ways leads to material dialogues the second to formal ones

If we are willing to accept something like *material truth*, then we can think that the grounds upon which the atomic formulae depend are facts of the world and a grounded atomic formulae is a way to say that it is true. However, something more general might be thought too, such as true in virtue of some player independent ground. This is the basis on which the rule for material dialogues has been formulated

Rückert pointed out that the formal rule establishes a kind of game where one of the players must play without knowing what the antagonist's justifications of the atomic formulae are. Thus, according to this view, the passage to formal dialogues relates to the switch to some kind of games with incomplete information. Now, if the ultimate grounds of a dialogical thesis are atomic formulae and if this is implemented by the use of a formal rule, then the dialogues are in this sense necessarily asymmetric. Indeed, if both contenders were restricted by the formal rule no atomic formula can ever be uttered. Thus, we implement the formal rule by designing one player, called the *proponent*, whose utterances of atomic formulae are, at least, at the start of the dialogue restricted by this rule.

Apparently, the formal rule introduces an asymmetry in relation to the commitments of **O** and **P** particularly so in the case of the utterance of the conditional. Indeed, if **O** utters a conditional, then **P**'s commits him to a series of moves that must at the end be based on atomic moves of **O**. If it is **O** that challenges a conditional no such commitment will be triggered. But it would be a mistake to draw from this fact the conclusion that the local meaning of the conditional is not symmetric. The very point of player independence is that it is a property of the meaning of the logical particles not of the dialogue as a whole where **P** is committed to the validity of the thesis. More precisely the asymmetry of the winning strategy is triggered by the semantic asymmetry of the formal rule. It is the possibility to isolate meaning (local and global) from validity commitments that allows dialogicians to speak of the symmetry of the logical constants and this prevents tonk-like operators from being introduced in the dialogical framework.

Examples:

In the following examples, the outer columns indicate the numerical label of the move, the inner columns state the number of a move targeted by an attack. Expressions are not listed following the order of the moves, but writing the defence on the same line as the corresponding attack, thus showing when a round is closed. Recall, from the particle rules, that the sign "—" signalises that there is no defence against the attack on a negation.

For the sake of simplicity we will assume the following rank choices:

O- ***r*** **:=** 1
P-***r'***:= 2

Ex. 1: Classical and intuitionistic rules

In the following dialogue played with classical structural rules **P**' move 4 answers **O**'s challenge in move 1, since **P**, according to the classical rule, is allowed to defend (once more) himself from the challenge in move 1. **P** states his defence in move 4 though, actually **O** did not repeat his challenge – we signalise this fact by inscribing the not repeated challenge between square brackets.

O			P		
				$p\vee\neg p$	0
1	$?_{\vee}$	0		$\neg p$	2
3	p	2		—	
[1]	[$?_{\vee}$]	[0]		p	4

Classical rules. **P** wins.

In the dialogue displayed below about the same thesis as before, **O** wins according to the intuitionistic structural rules because, after the challenger's last attack in move 3, **t**he intuitionist structural rule forbids **P** to defend himself (once more) from the challenge in move 1.

O			P		
				$p\vee\neg p$	0
1	$?_{\vee}$	0		$\neg p$	2
3	p	2		—	

Intuitionist rules. **O** wins.

2.3 Play level and Strategic Level

As mentioned above in the dialogical approach validity is defined via the notion of *winning strategy.*A systematic description of the winning strategies available for **P** in the context of the possible choices of **O** can be obtained from the following considerations[19]:

If **P** is to win against any choice of **O**, we will have to consider two main different situations, namely

- the dialogical situations in which **O** has uttered a complex formula, and
- those in which **P** has uttered a complex formula.

[19] Lorenzen 1978, 217-220. The relation with natural deduction has been recently worked out in Rahman/Clerbout/Keiff 2009, 301-336.

We call these main situations the **O**-cases and the **P**-cases, respectively. In both of these situations another distinction has to be examined:

(i) **P** wins by *choosing* between two possible challenges in the **O**-cases or between two possible defences in the **P**-cases, iff he can win with *at least one* of his choices.

(ii) When **O** can *choose* between two possible defences in the **O**-cases or between two possible challenges in the **P**-cases, **P** wins iff he can win *irrespective* of **O's** choices.

The description of the available strategies will yield a version of the semantic tableaux of Beth that become popular after the landmark work on semantic-trees by Raymond Smullyan (1968), where **O** stands for **T** (left-side) and **P** for **F** (right-side) and where situations of type **ii** (and not of type **i)** will lead to a branching-rule.

(**P**)-***Chooses***	(**O**)-***Chooses***
(**P**)$\alpha\vee\beta$ -------------------- <**O**?> (**P**)α <**O**?> (**P**)β *The expressions of the form <X…> constitute interrogative utterances*	(**P**) $\alpha\wedge\beta$ ------------------------------- <**O**?$\wedge$1> (**P**)α | <**P**?$\wedge$2> (**P**)β *The expressions of the form <X…> constitute interrogative utterances*
(**O**)$\alpha\wedge\beta$ --------------------- <**P**?$\wedge$1> (**O**)α <**P**?$\wedge$2> (**O**)β	(**O**) $\alpha\vee\beta$ ------------------------------- <**P**?> (**O**)α | <**P**?> (**O**)β
(**P**)$\alpha\rightarrow\beta$ -------------------- (**P**)α	(**O**) $\alpha\rightarrow\beta$ ------------------------------- (**P**) α

<table>
<tr><td>(O)β</td><td>(O) ? … | (O)β
(Opponent has the choice between counterattacking or defending)</td></tr>
<tr><td>No choice</td><td>No choice</td></tr>
<tr><td>(P) ¬α</td><td>(O) ¬α</td></tr>
<tr><td>------------------</td><td>---------------------</td></tr>
<tr><td>(O)α</td><td>(P) α</td></tr>
</table>

However, tableaux are not dialogues. The main point is that dialogues are built up bottom up, from local semantics to global semantics and from global semantics to validity. This establishes the priority of the play level over the winning-strategy-level. The levels are to be thought as defining an order. From the dialogical point of view, to set the meaning of the logical constants via validity is like trying to define the (meaning) moves of the king in the game of chess by the strategic rules of how to win a play. Neither semantic tableaux nor sequent calculus give priority to the play level. The point is not really that sequent calculus or tableaux do not have a play level, if with this we mean that one could not find the steps leading to the proof though there is one. What distinguishes the dialogical approach from other approaches is that in the other approaches – if there is something like a play level – the play level is ignored: the logical constants are defined via the rules that define validity.[20] The dialogical approach takes the play level as the level where meaning is set and on the basis of which validity rules should result. Within the dialogical approach, the more basic step of meaning at the play level is the setting of player-independent particle-rules (i.e. symmetric rules): the difference between **O (T)**-rules and the **P** (**F**)-rules results from the asymmetry introduced by the formal rule at the strategic level. These considerations lead us to tonk. One can build tableaux-rules for tonk and tonk-like operators but, from the dialogical point of view, they have no semantic underpinning.

[20] The point that other systems have also a play level has been stressed by Luca Tranchini in the workshop Workshop Amsterdam/Lille: *Dialogues and Games: Historical Roots and Contemporary Models*, 8-9 February 2010, Lille.

By a tree $\mathcal{T}$ we mean:

A set **T** of elements called nodes.
A function, ℓ, which assigns to each node t a natural number $\ell(t)$ called the label of t.($\ell := \mathrm{T} \rightarrow \mathbb{N}$).
A relation $t\mathcal{R}t'$ defined in **T** ($:= \mathcal{R} \subseteq \mathbf{T} \times \mathbf{T}$) to be read *$t$ is a predecessor of t'* or *t' is a successor of t*, such that:
(i) There is a unique node with label 0. This node, written t_0, is called the *root* of the tree.
(ii) For any $t \neq t_0$ there is a unique t' such that $t'\mathcal{R}t$.
(iii) For any nodes t, t' if $t\mathcal{R}t'$ then $\ell(t') = \ell(t) + 1$.

A *path* is any finite sequence of nodes that begin with the root and such that each term of the sequence is the immediate predecessor of the next one. An *end-node* is node with no successors.
A *branch* is a maximal path, i.e. a path that has an *end node* and this node is thus the last term of the path.

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