Identity and Quantification: Ibn Sīnā Self-Applied

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It is known that Aristotle decided not to develop the quantification of the predicate. A. Hasnawi showed in a recent thorough paper that Ibn Sīnā did.

In fact, Ibn Sīnā qualifies those propositions that carry a quantified predicate as *deviating* (*muḥarrafah*) propositions[1]. This terminology is linked to the Aristotelian subject-predicate structure assumed by Ibn Sīnā and has the consequence that the second quantification is absorbed by the predicate term. The clear differentiation between a quantified subject - that settles the domain of quantification – and a –predicative part – that builds a proposition over this domain corresponds structurally to what in the framework of constructive type theory is determined by the distinction between the *type set* and the type *prop(osition)*.

1 We follow here Hasnawi's [9] (pag. 323) translation of the term *muḥarrafah*. We will come back to this terminology further on.

Aristotle did not combine the logical analysis of quantification with his ontological theory of relations (n-adic predicates) or identity.

However,

1. in his syllogistic Ibn Sīnā makes use of syllogisms that require a logic of equality
2. Ibn Sīnā considered cases where quantification combines via identity with singular terms. - moreover these reflections provide the basis of his theory or numbers that is based on the interplay between *The One* and *the Many*.[2]

[2] Rahman/Salloum : *The One, the Many and Ibn Sīnā's Logic of Identity*. Paper in preparation.

If we combine Ibn Sīnā's metaphysical theory on identity and equivalence with his work on the quantification of the subject and the predicate a logic of equality seems to come out naturally.

Indeed, the interaction between quantification of the predicate and equality can be applied to Ibn Sīnā's own examples of syllogisms involving equivalence and identity.

In fact the paper could be seen as contributing to establish links between Ibn Sīnā's metaphysics and his logical work: links that have been discussed in relation to other topics by Paul Thom and Tony Street.

The present paper makes use of the formal instruments provided by the approach constructive type theory of Martin-Löf: The analysis deployed should justify this use by itself.

1 Introduction

Aristotle on quantified expressions

As pointed out by Göran Sundholm, since the work of Frege, quantified expressions are understood as standing for quantifiers that are intended to range over the universe of *all objects*. Hence, since all quantifications concern the same domain, there seems to be no practical or theoretical need to include explicit information concerning the domain of quantification in the quantifier-notation[3]: In such a setting the role of the predicates is to pick up from an all encompassing universe those subsets of objects appropriate for the analysis of the sentence at hand.

[3] Cf. G. Sundholm [14], and T. Fernando [5].

Such a strategy has as the side-effect that it liberates the logical form from the subject-predicate structure explicitly imposed on propositions by the main philosophical post-Aristotelian tradition.

However, the Fregean move is utterly unfaithful to the corresponding natural language expressions. It seems natural on one hand to attach to *every* a noun such as *every student* and *every philosopher*, and on the other, if someone asserts that

(1) *Some elephants are small*

it looks that we pick up from the restricted universe of elephants those that are small and *not* from the universal domain of objects those objects that are small and elephants. It may well be that the elephant in question is small (for an elephant), but even small elephants are big creatures in the animal kingdom. In fact 1, should be read as

(1*) *Some elephants are small elephants*

Michel Crubellier in a new French translation of the *Analytica* proposes to translate Aristotle's formulation of universal expressions such as (1) with the paraphrase

> *To be happy (or happiness) is the case for (applies to) all students*[4]

Crubellier's translation strongly suggests that Aristotle's formulation corresponds to a relation between terms – and that one of them, the subject, restricts the predicate (to be happy), to the domain of all the students[5]) - notice that *Every* and *Some* will always be attached to a noun (like in cardinal numbers).[6]. If we inverse the order of presentation of Crubellier's translation it yields the schema:

(Every Student) Happy

And

(Some Student) Happy

[4] Cf. Aristote [1].

[5] Cf. Aristotle [2], chapter 7 b12 .

[6] In a paper on the notion of number in Plato and Aristotle, M. Crubellier points out that for the ancient Greeks a number is always attached to a noun: two apples; two flowers; etc. The pure number of the mathematicians is, according to Aristotle, an abstract presentation that achieves generality – e.g. "two" abbreviates "two whatever objects" (Cf. M. Crubellier [7],). Perhaps, similarly we should think "every" as an abstraction of "every apple; "every flower". In other words, the "pure" logical form "every A" should be thought as an abstract presentation of universal expressions.

A crucial point is here the distinction is drawn between two forms of judgement involving *a is B,* namely:

$$a : \mathrm{B}$$

and

$$\mathrm{B}a \text{ is true}$$

The first concerns the relation between an element and a set and the second asserts a proposition.

This distinction is already present in Plato. Indeed, according to the beautiful analysis of Lorenz and Mittelstrass (1967) of the *Cratylus*, Plato points out two basic different acts of predication, namely,

stating (λέΥειν) and

naming (ὀνομάζειν).

The latter one amounts to the act of subsuming one individual under a concept and the first establishes a true proposition. *Naming* is about correctness: one individual reveals the concept it instantiates if the naming is correct (*names reveal objects for what they are*:

Names, i.e. predicates, are tools with which we distinguish objects from each other. To name objects or to let an individual fall under some concept is on the other hand the means to state something about objects, i.e. to teach and to learn about objects, as Plato prefers to say.

> […] *whereas only ' correct' names reveal objects for what they are (Crat. 422d), i.e. place individuals under an appropriate concept.* (Lorenz/Mittelstrass 1967, p. 7)

Stating is about the truth of the proposition that results of this kind of predication act. If an individual is indeed an element of the adequate type subset separated by the predicate at stake, the associated sentence is true.

> *Therefore, in Plato's terminology, a name is correct or reveals an object, if the associated elementary sentence is true, and incorrect if the associated elementary sentence is false.* (Lorenz/Mittelstrass (1967, p.8).

In the context of our own reconstruction *naming* (ὀνομάζειν) corresponds to the assertion that an individual is an element of a given set, that is it involves judgements of the form

b : A,

and *stating* (λέγειν) corresponds to building a proposition given the adequate elements of a set, that is

B*b* (where *b* : A and Bx becomes a proposition if the x are substituted by an element of the set A) .

Accordingly, the following notation stresses the two-term structure of the corresponding logical forms:

(*Every Elephant) Small* (*Every Elephant) not Small*

Yield:

$(\forall x : E)Sx$, (10) $(\forall x : E){\sim}Sx$,

And

(*Some Elephants) Small* (*Some Elephants) not Small*

Yield:

$(\exists x : E)Sx$ $(\exists x : E){\sim}Sx$

Notice that this notation also stresses the same two terms structure of the logical form for *every* proposition (singular, universal, negative or affirmative) but also

- The restricted quantification (the happy individuals are picked up from the set of students and not from the universal domain)
- The, to make use of the medieval terminology, *distribution* of the subject (the domain is universally understood) in 10, i.e. the domain of objects that are students is the whole domain of quantification; and is particular (or *undistributed*) in 11, i.e. the domain of objects that are students is not empty)

Aristotle did not apply his logical analysis of quantification either to relations (n-adic predicates) or identity. Ibn Sīnā considered the latter.

Ibn Sīnā considered propositions and syllogisms where quantification combines via equality with singular terms.

On our view; this is a natural development of the quantification of both, the subject and the predicate.

Moreover these reflections provide the basis of his theory or numbers that is based on the interplay between *The One* and *the Many*.[7].

Certainly, as pointed out by Ahmed Hasnawi[8], Aristotle discusses the case of relations and unity in many places of his work, such as in the books *delta* and *iota* of his *Methaphysics*. Moreover Ibn Sīnā's distinctions of unity in Genus, Species and by Accident that we discuss in the last part of the present paper are Aristotelian.

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[7] Rahman/Salloum : *The One, the Many and Ibn Sīnā's Logic of Identity*. Paper in preparation.
[8] Personal communication of Hasnawi.

However, and this is one main claim of our paper, it is Ibn Sīnā's quantification of the predicate, that allows to introduce these distinctions in the object language and prefigure; perhaps for the first time a logic with an identity predicate and an equivalence relation.

2 Ibn Sīnā and the Quantification of the Predicate

As already mentioned, the idea that quantifier expressions signify second-order relations does not seem to have been picked up by the mainstream of Aristotle's medieval followers. Instead they stressed the fact that the two terms have different status: the *Subject* and the *Predicate*.

Moreover, many of the logicians of the middle-ages avoided relations in general, first-order and second order – particularly so in the context of building syllogisms. Thus, syllogisms were based on monadic predicates and the logic of monadic predicates does not naturally lead to plural quantification.

However, Ibn Sīnā devotes two chapters of *al-ʿIbāra* to the quantification of the predicate. *Al-ʿIbāra* is the third book of the logical collection of his philosophical encyclopaedia entitled *al-Shifā'* (The Cure). It is in these texts that Ibn Sīnā shows that his approach to quantification is very close to the subject-predicate analysis of Aristotle mentioned above.

The understanding of quantification as based on a two-terms structure allows Ibn Sīnā both, to study double quantification and defend its study from detractors. This has been made apparent in the excellent paper of Ahmad Hasnawi [9] on Ibn Sīnā's double quantification – a paper that by the way contains an English translation of these two chapters, the first in any language. Indeed, one of the main objections against the quantification of the second term (i.e; the medieval predicate) is that, because of the possibility of a negation in the scope of the second quantifier the expressed propositions could not anymore said to be either affirmative or negative (take *Some man is not every animal.* Does this express an affirmative or a negative proposition?). Hasnawi sums up extremely well Ibn Sīna's position:

> *Avicenna upholds here a radical point of view: according to him the predicate in D[ouble]Q[uantificatied] P[roposition]s is constituted by the quantifier plus the initial predicate, which form together a unit. So a proposition which has the form of a normal affirmative proposition will keep this quality even though a negative quantifier has been prefixed to its predicate. The quantifier of the predicate is conceived of as a predicate-forming operator on predicates: it generates new predicates from previous ones by attaching a quantifier to them*.[9]

It very much looks as if Ibn Sīnā thinks of the quantification of the second term as building a new predicate that transfers the truth to the relation between the subject and the new *sum* – and this *deviates* of the *normal* use of quantification – since the second quantifier is seen here as a predicate.

[9] Hasnawi [9], pp. 304-306.

Let us once more quote Hasnawi and Ibn Sīnā himself:

To designate propositions with a quantified predicate Avicenna uses a seemingly technical term which is, as far as I know, unique to him in this context Such propositions are said to be munḥarifāt, which I translate as "deviating". Avicenna uses in other contexts words deriving from the root ḤRF to signify 1) that the illocutionary force of a proposition is changed or 2) that the truth-value of a proposition is suspended by the introduction of a "hypothetical particle".

He describes the first situation as follows: "The signification which is desired for itself (and not to provoke a reaction of the interlocutor) is [that of] the assertions (akhbār, or perhaps ikhbār, that is the act of asserting), used either normally ('alā wajhihā), or deviating (muḥarrafah) as is the case with wish and astonishment, for they reduce to the assertion(s)" (al-ʿIbāra: 31,10-11). He describes the second situation as following: "The unity of hypothetical propositions is due to the hypothetical link (ribāṭ al-sharṭ), which, when joined to the antecedent […], renders it deviating (ḥarrafahu), by making it neither true nor false." (al-ʿIbāra, 33,16-34,1). Whether 1) is reducible to 2) is not explicitly stated by Avicenna. The same semantic core, namely that a clause added to a proposition or to a part of it, makes the proposition deviate from its normal functioning, is present in the description Avicenna gives of propositions with a quantified predicate as munḥarifāt: "If you try then to add a quantifier, the proposition will be deviate (inḥarafat) : the predicate will no longer be a predicate, but rather it will become part of the predicate. ***The consideration of the truth will thus be transferred to the relation [65] which occurs between this sum and the subject. That is why these propositions were called deviating****" (al-ʿIbāra: 64, 17- 65, 1).*[10]

[10] Hasnawi [9], note 5, p. 323.

Ibn Sīnā analysis seems to blur the distinction between affirmative and negative propositions in the case of quantification of the predicate. Perhaps, because Ibn Sīnā is rather thinking in judgements – but if this were the case then this should also apply to judgements with only one quantifier. Be this as it may, Ibn Sīnā's double quantification allows the following analysis of dyadic predicates:

$$(\forall x : S)\ ((\exists y : P)\ Rxy)$$

In some way we might say that the second quantification deviating since it part of the second term of the whole expression.

Unfortunately this does not work for the identity predicate since

$$(\forall x : S) ((\exists y : P) x=y)$$

establishes the identity between objects of different domains (set types)

If we accept the preceding analysis the following seems to be adequate:

$$(\forall z : \{x: D) \mid Sx \} ((\exists w: \{(x : D) \mid Px\} \; p(z) = p\,(w))$$

Where *p(z)* is the left projection of (x : D) |Sx }, that is, those elements x of the domain D, such that the right projection *q(z)* witnesses the proposition *that those x are S.*

In other words : *All S are some P* (roughly) reads: *For all those z of D that are S there is at least one w of D that is P such that they are the very same object.*

Now, can we justify this form as rendering justice to Sina's analysis?

This will take us to the discussions developed in the rest of the paper

3 Identity, Equivalance and Quantification: Ibn Sīnā self-applied

The discussions on the *One* and *Many* are ubiquitous in the work of Ibn Sīnā and they share the features of many Neo-Platonist who, quite often, attempt to combine the Platonic with the Aristotelian Metaphysics. In fact these two notions set the basis for Ibn Sīnā's theory of numbers. A salient sense of the notion of *One* is, according to our author, identity developed in the first chapter on *Metaphysics* of the work the *Book of Science* and in the second chapter of the third book of *the Metaphysics of the Shifā*[11].

[11] *Le Livre de science, Métaphysique*; [3], p. 121-125 ; *La Métaphysique du Shifā'*, Livre 3, Chapitre 2, [5], p. 160-164

In this context, Ibn Sīnā distinguishes[12]:

- *One* in its nature: in no aspect multiplicity is involved, such as, God and the geometrical point.
- *One* in an aspect and multiple in another aspect. This One is said of specific things either according to accident or according to essence.

The first aspect of the *One*, the *One in nature*, seems to be related to the unity of an object. Perhaps even with the very concept of existence of an object.

The second aspect of the *One* that appears when we say that *One* is according to accident[13] amounts to say that something – i.e. when (a name of a) property or (name of an) individual - accompanies another (name of a) property or (name of an) individual, and both are the same.

والواحد بالعرض هو أن يقال في شيء يقارن شيئاً آخر، أنهما هو الآخر، وأما واحد. وذلك إما موضوع ومحمول عرضي، كقولنا: إن زيداً وابن عبد الله واحد، وإن زيداً والطبيب واحد؛ وإما محمولان موضوع، كقولنا: الطبيب هو وابن عبد الله واحد، إذ عرض أن كان شيء واحد طبيباً وابن عبد الله؛ أو موضوعان في محمول واحد عرضي، كقولنا: الثلج والجص واحد، أي في البياض، إذ قد عرض أن حمل عليهما عرض واحد[14]

12 *Le Livre de science, Métaphysique*, [3], p. 121-125 ; *La Métaphysique du Shifā'*, [5], p. 160-164.
13*La Métaphysique du Shifā'*, Book 3, Chapter 2, [5], p. 160.
[14] Avicenna, *Metafisica, La scienza dell cose divine* [6], p. 228. French translation in *La Métaphysique du Shifā'*, Livre 3, Chapitre 2, [5], p. 160.

This second sense (unity by accidens or essence) is the one that builds Ibn Sīnā's theory of equality and that very naturally combines with his quantification of the predicate.

Moreover the interaction between quantification of the predicate and equality can apply to Ibn Sīnā's own examples of syllogisms involving equivalence and identity and equivalence.

Indeed, Ibn Sīnā discusses transitivity of the equivalence relation in *Qiyās* i.6 (translation of W. Hodges):

> *Thus when you say C is equal to B and B is equal to D, so C is equal to D.*

Moreover in his *Autobiography* our author claims that he has reconstructed all the inference steps of the Euclides geometry. This certainly requires a profuse use of a logic of equality.

There seem to be quite a number of syllogisms Ibn Sīnā's work in logic involving identity and equivalence. They have not been all compiled yet but let us mention two that are very close to the examples of his Metaphysics, that we will discuss below:

At *Qiyās* 472.15f we find
:

> *Zayd is this person sitting down, and*
> *this person sitting down is white*
> *So, Zayd is white.*

The syllogism discussed at *Qiyās* 488.10 is probably intended to be read with identities too:

> *Pleasure is B.*
> *B is the good.*
> *Therefore pleasure is the good.*

In our fact Ibn Sīnā distinguished three cases of unity by accidents:

First case. A subject and an accidental predicate such as

Zayd and the physician are one or
Zayd and Ibn ’Abdallah are one.

Second case. Two predicates of a same subject such as:

The physician and Ibn ’Abdallah are one.

Third case. Two substances and an accidental predicate such as:

Snow and camphor are one in being white.

According to our proposal discussed in the previous chapter it seems to be natural to approach the notion of unity by means of the interplay between equality and restricted quantification. This assumes quantification of both the subject and the predicate[15].

Certainly, a different reconstruction, as the one displayed below is possible, assuming that " one" is ambiguous between

Identity: *Zayd and Abdallah are one* and

Copula: *Zayd and the physician are one*

15Certainly, the approach is anachronistic in the sense that we use quantifiers and set theory. However, strictly speaking what we need is some kind of domain of discourse and quantification expressions.

This has the consequence that the notion of one is ambiguous between copula and equality and that the unity-feature expressed in the natural language sentence disappears (at least explicitly) in the formalization

This even gets more patent in the following examples of Ibn Sīnā:

Snow and camphor are one in (relation to) being white

As

Everything that is snow and everything that is camphor is white, So let me consider that there is no real ambiguity copula-equality ambiguity and thus the first case:

(i) *Zayd and the physician are one*

Is different from *Zayd is a physician*

Moreover, I will assume here that our author is making use of both *Identity as a predicate* (*predicative identity*) and as an *equivalence relation* between objects. A distinction for which Constructive Type theory provides a precise frame: The first builds objects of the type propostion the second is defined within a set.

3.1 The Identitiy Predicate

According to Ibn Sīnā, a singular term is such that it is impossible for the mind to make it common to many items[16]. Such a term when used unambiguously indicates "the self of that which is ostensively designated", which self belongs uniquely to this designated object.

> **إما أن يكون معناه بحيث يمتنع في الذهن إيقاع الشركة فيه، أعنى في المحصل الواحد المقصود به، كقولنا زيد، فإن لفظ زيد؛ وإن كان قد يشترك فيه كثيرون، فإنما يشتركون من حيث المسموع؛ وأما معناه الواحد فيستحيل أن يجعل واحد منه مشتركا فيه؛ فإن الواحد من معانيه هو ذات المشار إليه، وذات هذا المشار إليه يمتنع في الذهن أن يجعل لغيره[17].**

16 *Al- Shifā'*, *Al-Madkhahl* [4], p. 26-27.
[17] *Al- Shifā'*, *Al-Madkhahl* [4], p. 26-27.

Now predicative identity does not necessarily assume always uniqueness. In fact I think, at least this is my proposal; that the examples of our author assume uniqueness.

Accordingly, we have the identity between an individual constant and a definite description (where we use the notation for *there is exactly one*):

(i') *There is exactly one (=:∃!) physician (=:P) who is the same as that man called Zayd (=:k_1)*:

$$(\exists ! x: M)\ Px \wedge (k_1 : M) \wedge\ x = k_1$$

Or in a more compact form

$$(\exists! z : \{ (x : M) \mid Px \})\ p(z)=k_1\ (k_1 : M)$$

Where *p(z)* is the left projection of (x : M) |Px }, that is, the only element x of M, such that the right projection *q(z)* witnesses the proposition *that he is a physician* applies).

In other words: there is exactly one z that is an element of those men that are physicians and this z is that same as the individual called Zayd.

The second example deploys *Ibn ’Abdallah* as an accidental predicate: *son of Abdallah* (=:I).

(ii) *Zayd and Ibn ’Abdallah are one*

That reads assuming uniqueness:

The only son of Abdallah is the same as that man called Zayd (=:k_1*):*

This yields

$(\exists! x : M)\ Ix \wedge (k_1 : M) \wedge x=k_1$

Or in a more compact form

$(\exists! z : \{ (x : M) \mid Ix \})\ p(z) = k_1\ (k_1 : M)$

Where *p(z)* is the left projection of (x : M) |Ix }, that is, the only element x of M, such that the right projection *q(z)* witnesses the proposition *that he is the sons of Abdallah* applies)

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The example of the Quiyas 472.15f mentioned above:

Zayd is this person sitting down, and
this person sitting down is white
So, Zayd is white.

Shows that Ibn Sīnā makes use of an instance of the following rule of substitution:

k_1 : M (i.e., Zayd : M)
k_2 : M, (k_2 : { x : M | *Sittingdown*(x) }
(it is true that) $k_1 = k_2$
(were the identity predicate has been defined in M)
Wk_2

Wk_1

Notice that this witnesses the interrelation between his metaphysical theory of equality and his logical studies, but in the other direction: that is, here it is the use of substitution in a syllogism that, so to say, completes Ibn Sīnā's metaphysical theory of an identity predicate.

The next example seems to deal with two definite descriptions:

(iii) *The physician and Ibn 'Abdallah are one*

The two predicates at stake are *being a Physician* and being *son of Abdallah* . Deploying the same notational conventions as before, and assuming once more uniqueness, we have the following reconstruction that stresses Ibn Sīnā's point mentioned above that the quantification of the predicate builds up a new second more complex predicate:

$$(\exists! z : \{ (x: M) \mid Ix \}) (\exists! w : \{ (y: M) \mid Iy \}) (p(z) = p(w))$$

3.2 *One* as an Equivalence Class: *One* in Accidents, Species and Genus

Let us recall the third case:

Snow and camphor are one in being white.

In relation to this case it looks as if Ibn Sīnā is defining equivalence classes.

In fact, our author speaks of two subjects, c*amphor* and *snow*, to which the same predicate to *be white* applies. This is a kind of higher-order language. Let us first assume such a language where the denotation of k_j is the property of *being snow* and k_j the property of *being camphor*. Then we could formulate the preceeding as:

(iv) $camphor \approx snow : \mathrm{W}$

(c*amphor* and *snow* are equivalent objects in the set W of white things)

It is not clear if our author, who is purported to have some Platonist inclinations, intended to speak of properties or mass-expressions as being *camphor* or *being snow* as elements of a set,or if he rather assumes that the equivalence relation exemplified above should be reduced to a lower order one.

For our purposes it is in principle enough to study the case of objects living it open if they are or not of higher order.

In this context, the example of the transitivity of the equivalence relation (*Qiyās* i.6) and the syllogism (*Qiyās* 488.10) is linked this time to his theory of the equivalence relation. Indeed, the following

Pleasure is B.
B is the good.
Therefore pleasure is the good.

can be read as an instance of transitivity :

$k_1 \approx k_2 : D$

$k_2 \approx k_3 : D$

$k_1 \approx k_3 : D$

In analogy to his syllogism involving an identity predicate it seems reasonable to assume that Ibn Sīnā would also make use of a substitution rule for equivalences such as

$$\begin{array}{l} k_1 : D \\ k_2 : D, \\ k_1 \approx k_2 \\ Wk_1 \\ \hline Wk_2 \end{array}$$

In fact our author restricts the constitution of an equivalence class by the use of Aristotelian categories instead to leave it to the arbitrary choice of a property or properties. Ibn Sīnā includes in this sense the following cases :

لكن الواحد الذي بالذات، منه واحد بالجنس، ومنه واحد بالنوع وهو الواحد بالفصل، ومنه واحد بالمناسبة، ومنه واحد بالموضوع، ومنه واحد بالعد د[18]

a. One in gender [*wāhid biljns*] such as:
The human and the horse are one by animality.

Since we do not distinguish between higher and lower objects we have

$$human \approx horse : \text{ANIMAL}$$

[18] Avicenna, *Metafisica, La scienza dell cose divine* [6], p. 228. *La Métaphysique du Shifā'*, Book 3, Chapter 2, [5], p. 160.

b. One in species [*wāhid bilnw*[c]] such as:

Zayd and Amr are one by humanity.

$$Zayd \approx Amr : \{ (x: A) \mid Hx \})$$

(Zayd and Amir are equivalent objects in the class of all animals that are human)

c. One in relationship [*wāhid bilmunāsaba*] such as :

The relation of the sovereign to the city and the relation of the soul to the body are one.[19]

One way to read this example of equality is to see it as if the pairs (sovereign, city) and (soul, body) instantiate the relation
x governs y

$$(\textit{sovereign}, \textit{city}) \approx (\textit{body}, \textit{soul}) : X \text{ GOVERNS } Y$$

[19] Our author considers herewith the constitution of an equivalence class by means of the specification of a determinate relation. For example take the two place relation *x governs y*. In that case the following ordered pairs could be seen as indistinguishable <Caesar, Rome>, <Einstein, Einstein's body>.

d. One by subject [*wāhid bilmwdu*^{c}] such as :
White and softness are one, like sugar. In reality, they are two but they correspond to the same subject.

The idea is in a piece of sugar it not possible to separate whiteness of its softness;

Wk ≈ Sk : prop (provided k : SUGAR)

(the propositions *k is white* and *k is sweet* are equivalent objects in the domain of propositions – i.e. in the type of propositions *prop*), provided k is an element of the set SUGAR of pieces of sugar)

e. One by number [*wāhid bil^c^dad*].

It is important to recall here that Ibn Sīnā thought, as Aristotle did before, that numbers are properties of objects – a claim that lost popularity after Frege's incisive objections in his *Grundlagen der Arithmetic*.

According to this view, we can build two equivalent propositions that ascribe, say to the object k (a kind of mass object, such as a pile of stones) and to the object j (say, a pile of apples) the same numerical property, e.g. the numerical property of being 7:

$$Nk \approx Nj : \text{ (where Nx is a prop provided x : D,)}$$

(where N is the numerical property of *being 7* such that the proposition that *the pile k of stones has the property 7* and the proposition that *the pile j of apples has the propety seven* are elements of equivalence class built by the property of being seven in the domain of propositions – i.e. in the type of propositions *prop*)

We have not found yet syllogisms that indicate how our author implemented the according substitutions.

However, sometimes Ibn Sīnā seems to be thinking of numbers as expressing the cardinality of two sets. In this sense the sets A and B are *one* in relation to having the same cardinality. Thus our example becomes

$$A \approx B : \text{Cardinal-number-seven (provided } A : \text{set}, B : \text{set)}$$

CONCLUSION

These few pages of Ibn Sīnā suggest that his logic explorations explored new paths beyond the work of Aristotle and particularly so in relation to equality.

As already mentioned in our introduction, one the main claims of our paper, is that Ibn Sīnā's quantification of the predicate, rejected by Aristotle; allows to combine, perhaps for the first time a logical an analysis of quantification combined with a theory of equality.

One task that still needs to be worked out as it is the study of Ibn Sīnā's theory of the relation between identity and existence in the context of his overall philosophical view on mathematics and particularly in the context of his *theory of numbers*.

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